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Research Article

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Mass Transfer in Cylindrical and Spherical Coordinates and Analytical Solution of the Governing Equation by AYM Approach

M. R. Akbari[1*], Sara Akbari[2], Esmaeil Kalantari[3]

[1]Department of Civil Engineering and Chemical Engineering, Germany.*
[2]Department of Pharmaceutical Technical Assistant, Dr. Kurt Blindow Vocational School, Germany.
[3]Department of Chemical Engineering, Islamic Azad University, Ghaemshahr, Iran.

Abstract
In this section, we want to investigate and analyse analytical governing differential equations in the mass transfer in cylindrical and spherical coordinates. As we know, most chemical reactions or heat transfer are in to the cylindrical and spherical, so the analytical solution of nonlinear differential equations are important.

Mathematical formulation of the problem

We assume that a chemical reaction of the second order takes place in a cylindrical or spherical bed as follows:

$$A \to p \quad ; \quad rA := k\,C^2 \tag{1}$$

The nonlinear differential equation governing the cylindrical and spherical coordinates are as follows:

$$\frac{Da}{r^n}\frac{d}{dr}\left(r^n \frac{dC(r)}{dr}\right) - k\,C(r)^2 = 0 \;\Rightarrow\; \begin{cases} n=1 & \text{Cylindrical} \\ n=2 & \text{Spherical} \end{cases} \tag{2}$$

Boundary conditions:

$$\frac{dC(r)}{dr} = 0 \quad at\ r=0\,,\ \ C(r) = Cs \quad at\ r=R \tag{3}$$

The parameter (***Da***) is the mass diffusion coefficient and (***k***) is the rate of the chemical reaction and parameter (***C***) is the concentration of the reactants in the chemical reaction and (***R***) is the cylindrical or spherical radius (***Cs***) is the initial concentration of the reaction material *A*.

AYM solution process (Akbari Yasna's Method)

Output of the solution process by new approach ***AYM (Akbari Yasna's Method)*** for nonlinear differential equation Eqs.(2), according to the boundary conditions Eqs.(3), the solution set of the non- differential equation is obtained as follows: **i)** In cylindrical coordinate **(*n=1*)** as follows:

$$C(r) := Y + \frac{1}{4}\frac{k\,Y^2}{Da}r^2 + \frac{1}{32}\frac{Y^3 k^2}{Da^2}r^4 \tag{4}$$

ii) In spherical coordinate **(*n=2*)** as follows:

$$C(r) := Y + \frac{1}{6}\frac{k\,Y^2}{Da}r^2 + \frac{1}{60}\frac{k^2 Y^3}{Da^2}r^4 \tag{5}$$

***Corresponding author:** M. R. Akbari, Department of Civil Engineering and Chemical Engineering, Germany. Tel: 0049 17647181018, 0049 17647181058; Email: akbari_hamid46@yahoo.com; sarah.akbari96@yahoo.com

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Parameter Y is Yasna coefficient.
Since the real boundary condition is at point ($r=R$), we apply the boundary condition on the Eqs. (4,5) as follows:

$$BC: \quad u(0)=u1,\ u(L)=u2 \tag{6}$$

After applying the boundary condition Eq. (6), in to Eqs.(4,5), the following algebraic equations is obtained:
i) In cylindrical coordinate ($n=1$):

$$Y+\frac{1}{4}\frac{kY^2}{Da}R^2+\frac{1}{32}\frac{Y^3k^2}{Da^2}R^4=Cs \tag{7}$$

ii) In spherical coordinate ($n=2$):

$$Y+\frac{1}{6}\frac{kY^2}{Da}R^2+\frac{1}{60}\frac{k^2Y^3}{Da^2}R^4=Cs \tag{8}$$

Algebraic equation Eqs.(7,8), we can easily calculate the Yasna coefficient Y for cylindrical and spherical coordinates as follows:
i) In cylindrical coordinate **($n=1$):**

$$Y:=\frac{2^{4/3}}{3R^2k}[\psi]^{1/3}-\frac{8}{3}\frac{Da^2\,2^{2/3}}{R^2k(\psi)^{1/3}}-\frac{8}{3}\frac{Da}{R^2k} \tag{9}$$

$$\psi=Da^2\left(27\,Cs\,R^2k+3\sqrt{3}\sqrt{27Cs^2R^4k^2+80\,Cs\,Da\,R^2k+64Da^2}+40\,Da\right) \tag{10}$$

ii) In spherical coordinate ***(n=2)*:**

$$Y:=\frac{1}{3}\frac{10^{1/3}(\psi)^{1/3}}{R^2k}-\frac{8}{3}\frac{Da^2\,10^{2/3}}{R^2k(\psi)^{1/3}}-\frac{10}{3}\frac{Da}{R^2k} \tag{11}$$

$$\psi=Da^2\left(81\,Cs\,R^2k+170\,Da+9\sqrt{81Cs^2R^4k^2+340\,Cs\,Da\,R^2k+420\,Da^2}\right) \tag{12}$$

By substituting the values of Y Eqs. (9-11) (Yasna coefficient) into Eqs.(4,5), the answer to the nonlinear differential equation Eq.(2) is obtained. By selecting the physical values at below:

$$Cs:=0.3;\ Da:=0.01;\ k:=0.5;\ R:=0.2 \tag{13}$$

Yasna coefficient for cylindrical coordinate of Eq. (9) is $Y:=0.263109829$ and for spherical coordinate Eq.(11) is:

$$Y:=0.273668724 \tag{14}$$

The solution is rewritten Eqs.(4,5) by applying physical values Eqs.(13) as follows:
i) In cylindrical coordinate ($n=1$):

$$C(r):=0.263109829+0.8653347765\,r^2+1.422988031\,r^4 \tag{15}$$

ii) In spherical coordinate ($n=2$):

$$C(r):=0.273668724+0.6241214208\,r^2+0.8540125642\,r^4 \tag{16}$$

Comparing the achieved solutions by Numerical Method order Runge-Kutta and *AYM* (Akbari Yasna's Method)

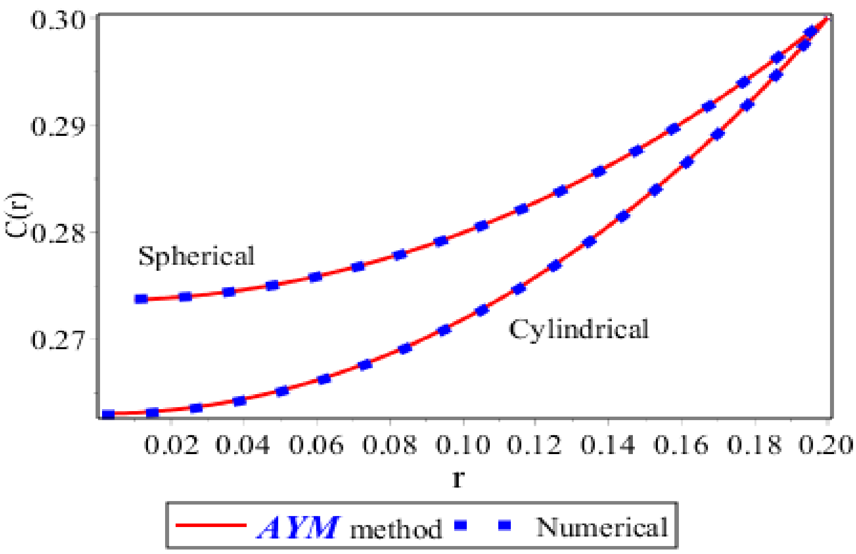

Figure 1: A comparison between *AYM* and Numerical solution for concentration for cylindrical and spherical.

We solve the previous differential equation at the general case

We consider the mass transfer differential equation for the case where the diffusion coefficient is a function of the concentration $Da = Do(1-\varepsilon C)$ in the cylindrical coordinate as follows:

$$\frac{1}{r}\frac{d}{dr}\left(Da\ r\ \frac{dC(r)}{dr}\right) - k\,C(r)^2 = 0\ ;\ Da = Do\,[1-\varepsilon\,C(r)] \quad (17)$$

Or as follows:

$$Do\,(1-\varepsilon\,C(r))\,\frac{d^2C(r)}{dr^2} - Do\,\varepsilon\left(\frac{dC(r)}{dr}\right)^2 + \frac{Do}{r}\,(1-\varepsilon\,C(r))\left(\frac{dC(r)}{dr}\right) - \frac{k}{r}\,C(r)^2 = 0 \quad (18)$$

And boundary conditions of:

$$\frac{dC(r)}{dr} = 0 \quad at\ r = 0\ ,\quad C(r) = Cs \quad at\ r = R \quad (19)$$

The solution of the mentioned problem by *AYM* will be obtained as follows:

By Applying Yasna coefficient Y in boundary conditions and by selecting the physical values at below:

$$Cs := 0.3;\ Do := 0.01;\ \varepsilon := 0.1;\ k := 0.5;\ R := 0.2 \quad (20)$$

Differential equation Eq. (17 or 18) is solved with AYM method and by applying of boundary conditions Eqs.(19) as follows:

$$C(r) := Y - \frac{125\,Y^2}{-10+Y}\,r^2 - \frac{78125\,Y^3}{Y^3 - 30\,Y^2 + 300\,Y - 1000}\,r^4 \quad (21)$$

We apply the boundary condition on the Eq. (21) as follows:

$$C(R) = Cs \quad (22)$$

After applying the boundary condition Eq. (22), in to Eq. (21), the following algebraic equations is obtained:

$$Y - \frac{5.00\,Y^2}{-10+Y} - \frac{125.0000\,Y^3}{Y^3 - 30\,Y^2 + 300\,Y - 1000} = 0.3 \quad (23)$$

In the algebraic equation Eq. (23), we can easily calculate the Yasna coefficient Y as follows:

$$Y := 0.2622459545 \quad (24)$$

By substituting the values of Y Eq. (24) (Yasna coefficient) in Eqs. (24), the answer to the nonlinear differential equation Eq.(17or 18) is obtained as follows:

$$C(r) := 0.2622459545 + 0.8828131765\,r^2 + 1.525949002\,r^4 \quad (25)$$

Comparing the achieved solutions by Numerical Method order Runge-Kutta and *AYM* (Akbari Yasna's Method)

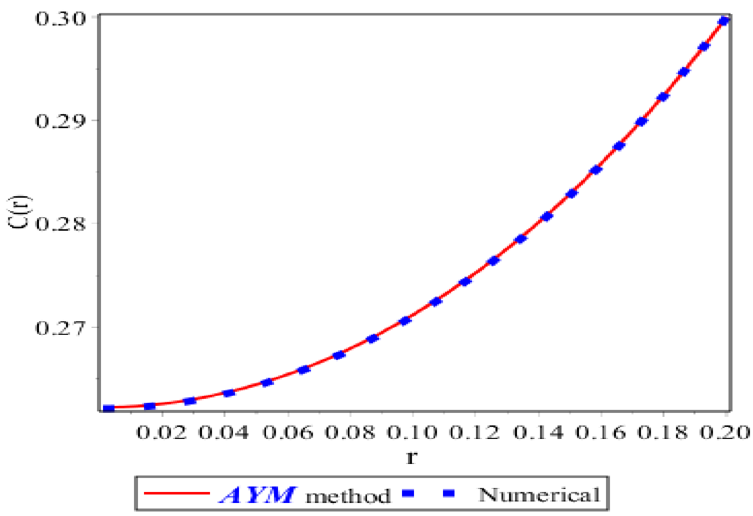

Figure 2: A comparison between *AYM* and Numerical solution for concentration for cylindrical.

Conclusions

In this article,we proved that with this new method, all kinds of complex practical problems related to nonlinear differential equations in the design chemical reactors can be easily solved analytically. Obviously,most of the phenomena in dynamics and aerodynamics are nonlinear, so it is quite difficult to study and analyze nonlinear mathematical equations in this area,also we wanted to demonstrate the strength, capability and flexibility of the new ***AYM*** method(Akbari-Yasna Method).This method is newly created and it can have high power in analytical solution of all kinds of industrial and practical problems in engineering fields and basic sciences for complex nonlinear differential equations.

Acknowledgment

History* of *AGM* , *ASM* , *AYM* , *AKLM* , *MR.AM* and *IAM* methods:** ***AGM (Akbari-Ganji Methods), ***ASM*** (Akbari-Sara's Method), ***AYM*** (Akbari-Yasna's Method) ***AKLM*** (Akbari Kalantari Leila Method), ***MR.AM*** (MohammadReza Akbari Method)and ***IAM*** (Integral Akbari Methods), have been invented mainly by Mohammadreza Akbari (M.R.Akbari) in order to provide a good service for researchers who are a pioneer in the field of nonlinear differential equations. ****AGM*** method Akbari Ganji method has been invented mainly by Mohammadreza Akbari in 2014. Noting that Prof. Davood Domairy Ganji co-operated in this project.
****ASM*** method (Akbari Sara's Method) has been created by Mohammadreza Akbari on 22 of August, in 2019.
****AYM*** method (Akbari Yasna's Method) has been created by Mohammadreza Akbari on 12 of April, in 2020.
****AKLM*** method (Akbari Kalantari Leila Method) has been created by Mohammadreza Akbari on 22 of August, in 2020.
****MR.AM*** method (MohammadReza Akbari Method) has been created by Mohammadreza Akbari on 10 of November, in 2020.
****IAM*** method (Integral Akbari Method) has been created by Mohammadreza Akbari on 5 of February, in 2021.

akbari_hamid46@yahoo.com

Reference

1. Book Nonlinear Dynamic in Engineering by Akbari-Ganji's MethodISBN-13: 978-1514401699, ISBN-10: 151440169X.
2. M. R. Akbari, Sara Akbari, Esmaeil Kalantari, 'Akbari-Ganji‟s method "AGM" to chemical reactor design for non-isothermal and non-adiabatic of mixed flow reactors'Journal of Chemical Engineering and Materials Science,Vol. 11(1), pp. 1-9, January-June 2020 DOI: 10.5897/JCEMS2018.0320Articles Number: 959C7AC63731 ISSN 2141-6605 Copyright © 2020 Author(s) retain the copyright of this article ,http://www.academicjournals.org/JCEMS.
3. M. R. Akbari, D. D. Ganji, M. Nimafar, 2014, "Significant progress in solution of nonlinear equations at displacement of structure and heat transfer extended surface by new AGM approach", Frontiers of Mechanical Engineering Journal, DOI: 10.1007/s11465-014-0313-y.
4. A. K. Rostami, M. R. Akbari, D. D. Ganji, S. Heydari, Investigating Jeffery-Hamel flow with high magnetic field and nanoparticle by HPM and AGM, Cent. Eur. J. Eng. 4(4) . 2014 .357-370, DOI: 10.2478/s13531-013-0175-9.
5. M. R. Akbari, D. D. Ganji, A. Majidian , A. R. Ahmadi, "Solving nonlinear differential equations of Vanderpol , Rayleigh and Duffing by AGM", Frontiers of Mechanical Engineering, April 2014 .
6. D.D. Ganji, M.R. Akbari, A.R. Goltabar, "Dynamic Vibration Analysis for Non-linear Partial Differential Equation of the Beam - columns with Shear Deformation and Rotary Inertia by AGM", Development and Applications of Oceanic Engineering (DAOE), ISSN Online: 2325-3762,2014.
7. M. R. Akbari, D.D.Ganji, A.R.Ahmadi, Sayyid H.Hashemi kachapi, "Analyzing the Nonlinear Vibrational wave differential equation for the simplified model of Tower Cranes by (AGM)", Frontiers of Mechanical Engineering March 2014, Volume 9, Issue 1,pp 58-70.
8. M. R. Akbari, M. Nimafar, D. D. Ganji, M. M. Akbarzade, "Scrutiny of non-linear differential equations Euler Bernoulli beam with large rotational deviation by AGM" Springer, December 2014 , (DOI) 10.1007/s11465-014-0316-8.
9. M. R. Akbari, Sara Akbari, Esmaeil Kalantari, "A Study about Exothermic Chemical Reactor by ASM Approach Strategy"Crimson Publishers Wings to the Research,Published, March 17, 2020.