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A Redefinition of The Heaviside Step Function and The Recursive Heaviside Step Function

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Abstract

In this paper, we redefine a new Heaviside step function which violating the Heaviside distribution by adding a Kronecker delta-like sequence to classical Heaviside sequence.
The new Heaviside step function at t=0, is given as a nonbinary positive number except 1 and 0. The recursive Heaviside step function redefined by using the new defined Heaviside step function, satisfies a multidimensional advection equation. The nonbinary positive number of the recursive Heaviside step function may describe a human mental state.

A new definition of Heaviside step function

Introduction

A numerical value of Heaviside step function(H(t)) at t=0 can be given as any positive nonbinary integer value. However, we show and assume that any positive nonbinary integer of Heaviside step function at t=0 can be expressed by limit of sequences. In this paper, we define a new Heaviside step function that has a positive nonbinary number at t=0 and review a recursive Heaviside step function introduced by Shin et al (2016) the first time.

Introduction of a Kronecker-like delta function
Let's start with a sequence described as below

$$\delta_L(x) = \lim_{n\to\infty} e^{-n^2x^2} \qquad (1)$$

Hereafter, we call an equation (1) as a Kronecker Delta-like function. One of many Heaviside sequences is expressed as

$$H(t) = \lim_{n\to\infty} \frac{1}{1+e^{-2nt}} \qquad (2)$$

A value of equation (2) at t=0 can be 1 or 0.5 or undefined(Kreyszig, 2012, Zauderer, 1998).
If we add equation (1) to equation (2), we can introduce a new Heaviside step function as $H_n(t) = \lim_{n\to\infty} \frac{1}{1+e^{-2nt}} +$

$$A \lim_{n\to\infty} e^{-n^2t^2} \qquad (3)$$

where A is an arbitrary positive nonbinary integer except 1 or 0.
Then, the value of new Heaviside step function can be assumed as any positive nonbinary number except 1 and 0.
A reason of why 0 cannot be defined as the value of the Heaviside step function(H(t)) at t=0, will be discussed in following section.

A new recursive Heaviside step function

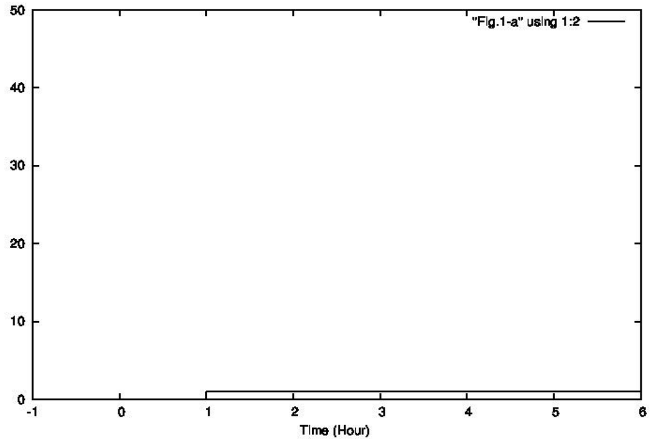
A recursive Heaviside step function was introduced. By Shin et al (2016), in order to describe mental state of human. However, their dilemma was that the mental state of human can be given as binary numbers (0 or 1) only. We can express one is alive(conscious) as 1 whereas we can express one is dead (in consciousness) as 0. If the value of the Heaviside step function at x=0 is the positive non binary integer except 0 or 1, the nonbinary positive number represent a registered memory at a given time when human sensors something through five organs.

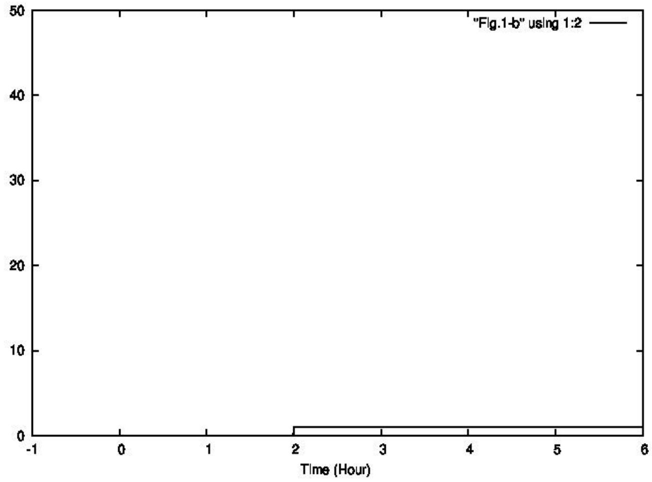
For example, let's consider recursive Heaviside step function expressed as below

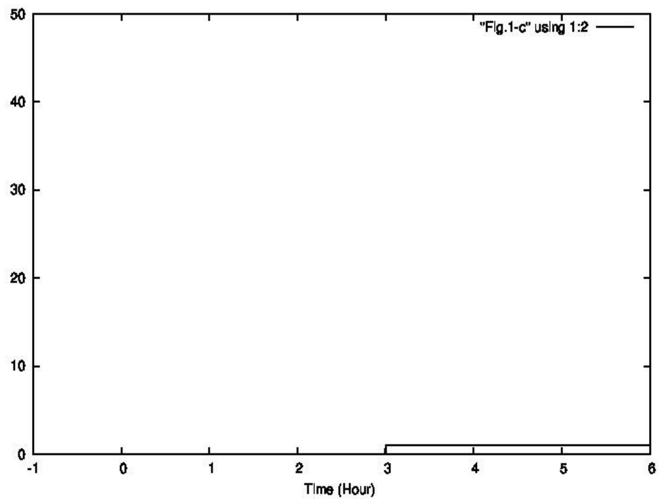
$$\mathrm{u(t)} = \mathrm{H(t-1+H(t-2+H(t-3+H\,(t-4+H(t-5)))))}$$
(4)-1
$$\mathrm{u(t)} = \mathrm{H(-t+1+H(-t+2+H(-t+3+H\,(-t+4+H(-t+5)))))}$$
(4)-2

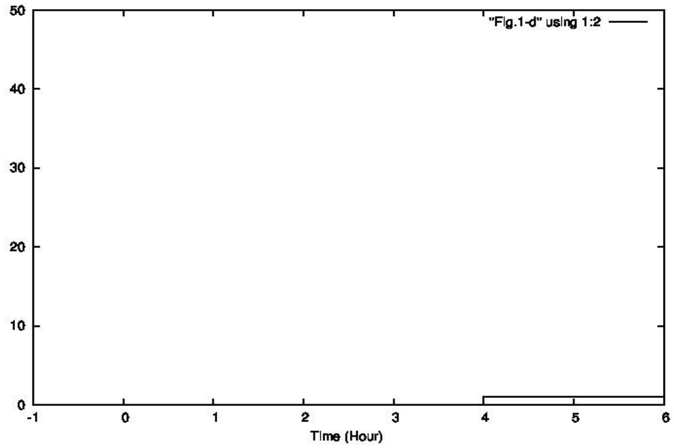
The difference between the recursive Heaviside step function suggested by Shin et al (2016) and equation (4)-1 and (4)-2 lie in a fact that + in equation (4) is replaced by multiplication.

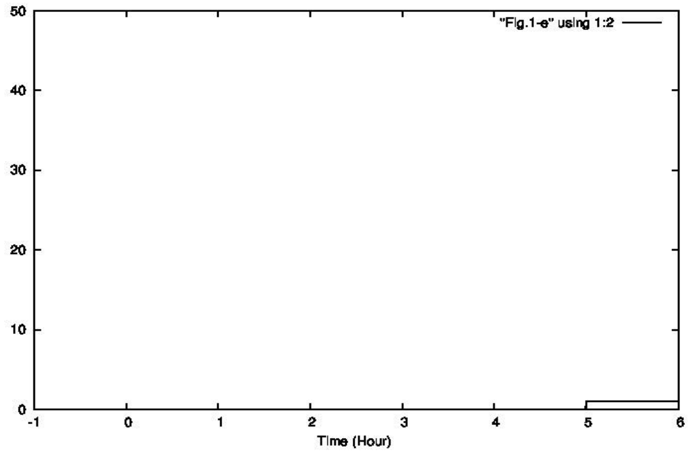
If we assume that the value of the Heaviside step function at t=0 is defined as 1, we can plot equation (4)-1 and (4)-2 as function of time, as shown in Figure (1-a), Figure(1-b), Figure(1-c), Figure(1-d), and Figure(1-e) as time flows. Note that time flows from right to left.

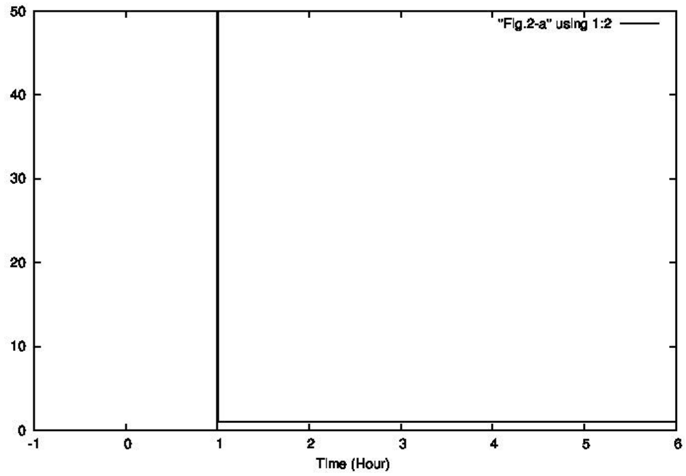


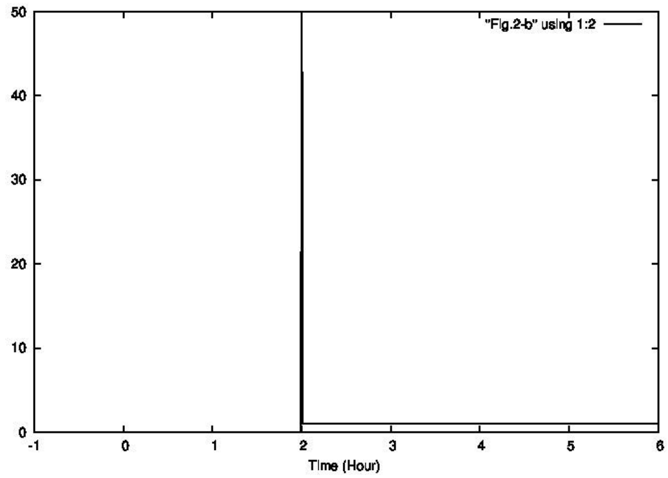


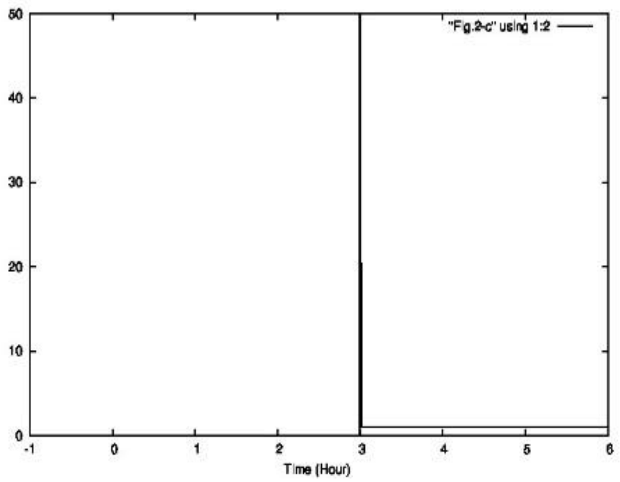


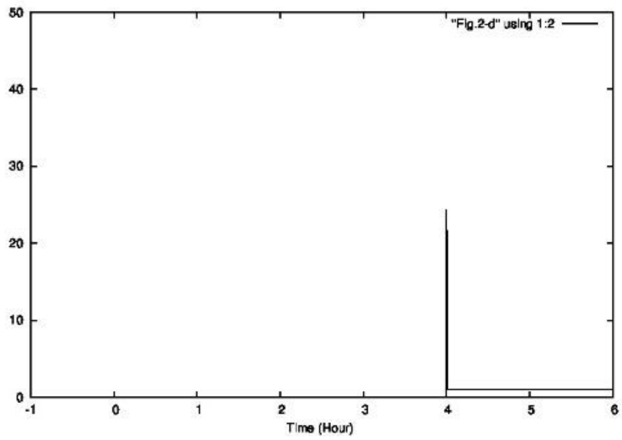


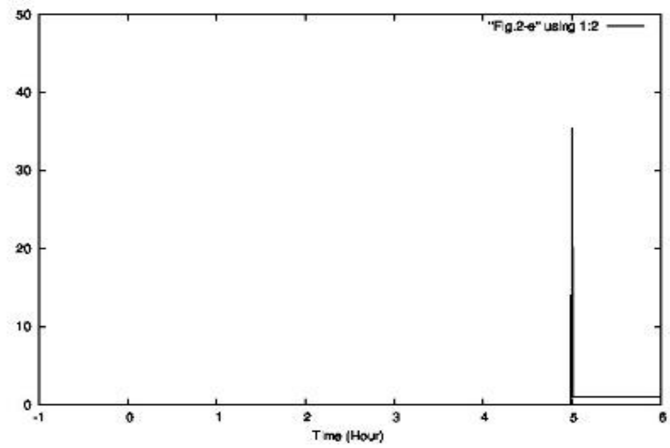












If the value of Heaviside step function is given as the arbitrary positive integer except 1, we can display as in Figure (2-a), Figure(2-b), Figure (2-c), Figure (2-d), Figure (2-d) and Figure(2-e).

It means that we can register an event experienced by human and store the event as a function of the recursive Heaviside step function as shown in Figure (2). However, as shown in Figures (1), if we define the value of the Heaviside step function as 1, we register same memory all the time. By adding delta-like sequence of equation (1) to the Heaviside step function (positive nonbinary integer at the Heaviside step function at t=0), we assume that a memory of the brain can be expressed by the recursive Heaviside step function.

However, we cannot make such storage device under modern technology. Multidimensional advection equation and the recursive Heaviside step function Shin et al (2016) introduced a multidimensional advection equation expressed as

$$\frac{\partial u}{\partial t} = -\frac{\partial u}{\partial \tau_1} - \frac{\partial u}{\partial \tau_2} - \frac{\partial u}{\partial \tau_3} - \frac{\partial u}{\partial \tau_4} - \cdots - \frac{\partial u}{\partial \tau_n} \qquad (5)$$

where t is time, τ_1 is current time, τ_i ,i=2,…,n is event registered past time.
By inspection, one of solutions of equation (5) is given as

$$U=H\,(t-\tau_1 + H(t-\tau_2 + H\,(t-\tau_3 + H(t-\tau_4 + H(t-\tau_5 \text{---}(6)\text{-}1$$
$$U=H\,(-t+\tau_1 + H(-t+\tau_2 + H\,(-t+\tau_3 + H(-t+\tau_4 + H(-t+\tau_5 \text{---}(6)\text{-}2$$

Conclusion

By adding the delta-like sequence to the Heaviside sequence, we redefined the Heaviside step function, which allows to express memory of the human brain. Furthermore, it does destroy a Heaviside distribution.
The shape of the recursive Heaviside step functions of each different person are equivalent to each other, therefore hiding
each other's registered different memory in the same shape of the recursive Heaviside step functions.

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