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Research Article

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Study of Unsteady Turbulent Plane-Parallel Pressure Flow in The Entrance Region

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Abstract
Irrigation plays an important role in the development of agriculture, especially in mountainous areas, which is provided by pumping stations. In the pipelines of such stations, the movement of water is often turbulent, and therefore its study is one of the main problems of ensuring the uninterrupted operation of irrigation systems. The study of turbulent unsteady plane-parallel flow at the entrance transition region was carried out taking into account the change of the kinematic coefficient of viscosity. The latter depending on the distance from the fixed wall was described by a linear law, which formulated a boundary problem. This is system of differential of continuum equations where the kinematic coefficient of viscosity was adopted as a system of starting equations in accordance with turbulent regimes of the flow. Within the framework of this problem, the initial equations were simplified and boundary conditions were defined. The solution to the boundary value problem was obtained by methods of integrating partial differential equations that satisfy all boundary conditions. Analytical solutions for this problem have been obtained, which make it possible to obtain regularities of velocity and pressure change in the direction of flow, at any point along the hydrodynamic entrance region and at any moment of time. Based on the general solutions of the problem, solutions were obtained for two cases. a) the velocity of the fluid entering the cylindrical tube is constant, b) the velocity of the entering fluid has a parabolic distribution. For the mentioned cases, computer-aided graphs of the velocity change in different cross-sections along the length of the entrance transition region and for any moment of time have been plotted. With the obtained composite graphs, the diagrams of the time-dependent velocity change along the entire length of the inlet transition zone have been constructed, which enables obtaining the flow velocity at any point of the cross-section and finding the length of the hydrodynamic entrance region.

Keywords: plane-parallel motion, inlet cross-section, unsteady turbulent motion, tangential turbulent stresses, velocity distribution

1. Introduction

The features inherent in the pressure system are such that current hydromechanical phenomena are often accompanied by unsteady movements of the working medium over time, causing changes in velocity, pressure and density of the medium at any point of the section. To study these changes, when considering practical issues, a non-stationary model of motion of the same magnitude is used as a calculation model, where the flow parameters are determined by the averaged values of the section.

Deviations of the averaged values from the hydromechanical parameters of the current cut points are taken into account using kinetic energy coefficients and the amount of motion. In this regard, one of the important issues of non-stationary motion is the study of the patterns of change of these coefficients, for which it is necessary to conduct structural studies of non-stationary motion.

Often, complex mathematical models have to be used to carry out these observations, and the results obtained turn out to be impractical for practical application. In view of this, in order to bring some clarity to the calculations, we proceed from a quasi-stationary model of motion. In this case, non-stationary motion is considered as a sequence of movements. At any given time, the average flow velocity is equated to the average speed of apparent stationary motion. By this, non-stationary motion is considered as a unity of stationary movements following each other. With such a model of unsteady motion, there is a linear relationship between the coefficient of friction resistance and the stresses formed during friction at the wall.

Such an assumption can give in reality as accurate results as the plot of the instantaneous velocity distribution coincides with the law of quadratic parabolas. However, experimental and theoretical results show that in reality, the plots of the instantaneous velocity distribution can be rigidly different from the law of quadratic parabolas, which is why, based on the law of quasi-stationary motion, energy losses cannot be determined by the friction stress formed at the wall. Depending on the law of change of instantaneous motion in the section, the friction stress formed at the wall also changes, which is why it will differ from the corresponding friction stress of quasi-stationary motion. In order to obtain a change in the friction stress formed when the wall is stationary during non-stationary motion, it is necessary to investigate the patterns of velocity changes in the real section.

So, the main task of studying any unsteady motion under pressure is the study of structural changes in the flow, which leads to the definition of the law of velocity change at any point in time at any point in the real section and, as a result, to the discussion of the uneven distribution of velocities, changes in the coefficients of the amount of motion, the distribution of friction stresses in the section, the assessment of individual data of energy loss.

In the transition areas of closed beds, there is a rearrangement of velocities, the particles near the stationary interior wall of a pipe perform a decelerating motion, and the particles near the axis of the pipe gain speed and perform an accelerating motion. The precise engineering of the fluid channels of mechanical equipment greatly depends on thorough investigation of processes running in the hydrodynamic entrance region. Investigations of hydrodynamic phenomena in transitional areas were mainly performed under conditions of stationary laminar motion, whereas in reality, laminar motion in short fluid channels is uncommon. Basically, the nature of the fluid flow is turbulent. Therefore, the study of hydrodynamic parameters' behavior of turbulent flow in transitional areas of closed beds is important, for its results can serve as a serious base for mechanical equipment designers. A transition zone is considered to be the section of velocity rearrangement in which the patterns of velocity distribution become equivalent to the patterns of the stabilized zone of closed beds. In case of non-stationary motion, the change of velocities occurs depending on time and length coordinates. To study hydrodynamic phenomena in the transition area many theoretical and approximate calculation methods have been developed. On the base of each calculation method lies conclusions related to the nature of the flow, with which theoretical research and summary of results are carried out. Often, these conclusions refer to a certain range of motion, which limits applicability of the obtained results.

2. Literature Review and Problem Statement

The patterns of changes in hydrodynamic parameters under conditions of accelerated flow of viscous liquid in the a round pipe's entrance region were analyzed. Velocity change graphs have been plotted for different time values [1].

The discrepancy between analytical and numerical solutions of unsteady laminar motion of a viscous fluid at the inlet section of a round pipe has been proven [2]. The existence of a maximum speed and its location have been experimentally proven. Various transient laminar movements have been studied [3]. A comparison of analytical and corresponding approximating solutions has been carried out, and recommendations for the use of these solutions in engineering practice have been proposed.

The study of transient processes when the wave speed varies along its length is a practical problem. An analytical solution has been developed for laminar flow with wave speed varying along the length [4]. A comparison of the obtained results with the results obtained by the characteristics method is presented. In [5], depending on the change in viscosity and pressure gradient, a solution was obtained for the hydrodynamic parameters of laminar flow in axisymmetric pipes. The general solution is obtained using the finite Hankel transform method.

In [6], the unsteady laminar flow of a viscous incompressible fluid at the inlet section of a round cylindrical pipe under general initial and boundary conditions was considered. Based on the research results, patterns of changes in speeds and pressure along the length of the transition section depending on time were revealed. Graphs of changes in the indicated physical quantities were constructed. A similar problem for stationary plane-parallel pressure motion is considered in [7].

At the site of sudden expansion of the section of the cylindrical tube, the patterns of changes in the hydrodynamic parameters of the viscous strait were studied in the case of general boundary conditions [8]. The length of the transition region, which depends on the initial velocity distribution in the inlet section, was estimated. In case of constant speed distribution, it is 0.174R*Re, and in case of parabolic distribution, it is 0.163R*Re.

An analysis of the linear stability of the flow in a pipe was carried out with a stepwise increase in flow rate from the steady initial flow [9]. A stepwise increase in flow causes non-periodic unsteady movements. The stability characteristics of an unsteady flow with stepwise changes in flow rate have been studied.

The results of experimental studies of the flow of sudden expansion of a round pipe are presented [10]. It was revealed that, depending on the Reynolds number, localized turbulent spots are formed behind the expansion. The mechanism of development of turbulent spots is explained.

Numerical study of oscillatory plane-parallel motion of a Newtonian fluid in the input region [11]. Based on the results of numerical data, a correlation was obtained to determine the length of the hydrodynamic entrance region. The study of turbulent fluid flow through sudden expansion channels has practical applications [12]. The flow characteristics of the recirculation zone in a sudden expansion pipe are analyzed.

Turbulence is the main cause of friction losses. The distortion of the velocity profile leads to the complete disappearance of turbulence. It has been proven that a return to laminar flow is achieved through an initial increase in turbulence intensity or a temporary increase in wall shear [13]. The unsteady motion of an incompressible fluid in a round pipe was studied with the coefficient of kinematic viscosity varying arbitrarily over time [14]. The results of the analytical solution are compared with the direct numerical solution of the momentum Eq., the difference is 1%. Experimental studies of the movement of a viscous fluid by sudden expansion were carried out. At moderate Reynolds numbers, experimental studies of the patterns of parameters of laminar motion of a viscous fluid during sudden expansion of the pipe diameter were carried out [15]. The condition behind the expansion of the formation of turbulent spots has been identified. A numerical study of the oscillatory laminar motion of an incompressible fluid in the inlet region of a plane-parallel channel was carried out. The development of axial velocity profiles and the required entry length were studied in the low Reynolds number regime [16]. The turbulent flow of liquid through sudden expansion channels has been studied. A sharp change in geometry leads to a sharp pressure drop and a recirculation zone is formed [17]. The flow characteristics in the recirculation zone in a pipe with a sudden expansion ratio of 1:2 at a Reynolds number of 20,000 are analyzed.

The mentioned studies mainly concern the interpretation of phenomena occurring at the pipe entrance site. However, the phenomena of rearrangement of hydrodynamic parameters also occur in other transitional sections of the pipe, about which there is little research. At the site of the sudden expansion of the section (D/d = 4), force lines were constructed using equations of motion of the flow of a plastic liquid poppy, numerical integration, and changes in velocity and pressure in the direction of the axis were determined [18]. In [19], numerical modeling of the incompressible fluid flow was carried out at the sites of sudden changes in the geometric dimensions of a fixed channel.

Using the method of magnetic resonance imaging, the authors were able to obtain quantitative estimates of velocity changes in the transition region [20]. Under conditions of sudden, symmetrical and asymmetric expansion of the section, quantitative estimates of the terms of the Neiva-Stokes equations were determined, as a result of which the obtained nonlinear inhomogeneous differential equations were integrated numerically [21]. The results of the integration were compared with the results of the experiments. In [22], extraordinary experimental studies were conducted in the field of sudden expansion of the incision. For this purpose, experimental equipment was built and sections of sudden expansion of the incision were tested in cases d/D=0.22;0.5;0.85. The research was carried out under conditions of Newtonian and non-Newtonian fluids.

New scaling expressions are proposed for all components of the Reynolds stress in the entire flow region of turbulent flows in channels and pipes [23]. Taking into account the second component, normal to the wall, it is proved that not all values of the turbulent stress approach the self-similar asymptotic state at the same rate as the Reynolds number increases, and the Reynolds tangential stress approaches faster than the tangential normal stress in the flow direction [24]. The reasons for this trend are explained by the appearance of wall vortices. The analysis of the logarithmic profile of the velocity distribution in a turbulent flow was carried out in [25].

3. The aims and objectives of the study

The purpose of the study is to identify patterns of changes in the hydrodynamic parameters of an incompressible fluid at the inlet section of plane-parallel pressure motion during unsteady turbulent motion.

To achieve this goal, the following tasks are solved:

- formulate a boundary value problem and develop a method for solving the boundary value problem and identify the pattern of changes in the hydrodynamic parameters of an unsteady turbulent flow at the inlet section of plane-parallel pressure motion,
- construct graphs of changes in axial velocities depending on time, identify conditions for determining the length of the inlet section of plane-parallel pressure motion during unsteady turbulent liquefaction.

4. Materials and methods

4. 1. Choosing a Calculation Scheme

The study of the passage section of the entrance section mainly concerns laminar movements. However, laminar motion is rare in practice. In engineering tasks, movement is usually turbulent. Therefore, the study of the patterns of movement in the transition areas at the entrance to cylindrical channels during turbulent movement is of important practical and theoretical interest.

The study of the movement patterns at the transition sites of the entrance shear of the fluid channels in the case of turbulent non-stationary motion has an important practical and theoretical interest. In a turbulent flow, frictional stresses arise between the liquid layers, the magnitude of which depends on the distance of the layer from the stationary wall. In the case of smooth parallel turbulent motion, frictional stresses arising between fluid layers, according to Boussenesque, are determined by the y coordinate from the stationary wall by the following Eq. [23]:

$$\tau = -\rho\varepsilon \frac{d\bar{u}}{dy} . \tag{1}$$

where ε is the kinematic coefficient of turbulent viscosity.

There are no studies in the literature on the kinematic coefficient of turbulent viscosity in non-stationary turbulent flow. Therefore, for the study of turbulent non-stationary motion currents, as a first approximation, let's use the assumptions made regarding stationary turbulent currents. In a turbulent stream of stationary motion, in the case of smooth parallel motion, according to the conclusion of Prandtl [26], the kinematic coefficient of turbulent viscosity, depending on the distance of the point from the stationary wall, changes according to the linear law

$$\varepsilon = n \cdot y. \tag{2}$$

The relationship of the turbulent viscosity kinematic coefficient (2) for stationary turbulent motions provides an exact match between theoretical and experimental studies [26]. This linear relationship in the case of stationary turbulent motion in live shear causes a logarithmic dependence of the distribution of velocities, which coincides with sufficient accuracy with the results of experimental studies [26]. It is assumed that the linear dependence of the kinematic viscosity coefficient can provide sufficient accuracy also for unsteady turbulent flows.

In the transition sections, the particle velocity changes in two directions: in the direction of the *oz* axis of motion and in the direction of the *oy* axis perpendicular to the axis of motion. Therefore, the average velocity of the turbulent current at the junction depends on the coordinates of the z and y points and time $\overline{U} = \overline{U}(z, y, t)$.

To solve the problem, the regularity of the velocity distribution in the input section is set as an arbitrary function $\overline{U} = \varphi(y)$. The mathematical model of the problem leads to the integration of the equations obtained by L. Prandtl [26] from the Navier-Stokes equations for the boundary layer.

4. 2. Statement of the problem and formulation of the system of differential

The Eq. of one-dimensional pressure motion of an incompressible fluid is expressed as [26].

$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial z} + \frac{\partial \tau}{\partial y}. \tag{3}$$

In the transition region, the velocity of the particle changes in two directions: in the direction of the z axis of motion and in the direction of the y axis perpendicular to the axis of motion. Therefore, the average speed of the turbulent current in the transition area depends on the z and y coordinates of the point and time: $u = u(z, y, t)$. To solve the problem, the speed distribution pattern in the inlet cross-section is given in the form of an arbitrary function $\overline{U} = \varphi(y)$.

The mathematical model of the problem leads to the integration of Eq. (3), which after the approximation of the nonlinear term [27] takes the following form

$$\frac{\partial u}{\partial t} + \mathrm{U}_o \cdot \frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{1}{\rho}\frac{\partial \tau}{\partial y}, \tag{4}$$

where y is calculated from the stationary wall of the pipe. Taking into account relations (1), (2), the last Eq. will take the following form:

$$\frac{\partial u}{\partial t} + \mathrm{U}_o \cdot \frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} - n\left(y\frac{\partial^2 u}{\partial \mathrm{y}^2} + \frac{\partial u}{\partial y}\right) \tag{5}$$

$$0 \le y \le h$$

The internal boundary conditions of Eq. (5) will be

$$u(z,0,t) = 0, \quad u(0,y,t)|= \varphi(y), \quad \frac{\partial u(z,0,t)}{\partial y} = 0 \tag{6}$$

The Eq. of indivisibility in this case will be

$$\frac{\partial u}{\partial z} + \frac{\partial u}{\partial y} = 0, \tag{7}$$

where U_0 is the characteristic velocity of the cross-section, which is equal to the average velocity of the effective cross-section

$$U_o = \frac{1}{h}\int_0^{+h} \varphi(y)dy : \tag{8}$$

It is assumed that the constant change in pressure gradient depends only on the z-coordinate of the path. It means that the pressures at all points of the live section have the same values, therefore

$$-\frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial z} = f(z). \tag{9}$$

Considering Eq.(9), Eq.(5) will take the following form

$$\frac{\partial u}{\partial t} + U_o \frac{\partial u}{\partial z} = f(z) - n\left(y\frac{\partial^2 u}{\partial \mathrm{y}^2} + \frac{\partial u}{\partial y}\right): \tag{10}$$

Substituting dimensionless variables

$$U(\sigma, x, \tau) = \frac{u}{U_0}, \quad \frac{y}{h} = x, \quad \frac{P}{P_0} = p, \quad \frac{z}{h} = \sigma, \quad \frac{t}{t_0} = \tau \tag{11}$$

into Eq.(5) and boundary conditions (6) we get

$$\frac{\partial U}{\partial \tau} + \alpha \frac{\partial U}{\partial \sigma} = -F(\sigma, \tau) - \beta_0 \left(\frac{\partial U}{\partial x} + x\frac{\partial^2 U}{\partial x^2}\right) \tag{12}$$

$$U(\sigma, 0, \tau) = 0, \tag{13}$$

$$U(0,x,0)=\phi(x),\quad \text{when}\quad (0<x\leq 1), \tag{14}$$

$$\frac{\partial U(\sigma,x,t)}{\partial x}\Big|_{x=1}=0. \tag{15}$$

where $$F(\sigma,\tau)=\frac{t_0 P_0}{h\rho U_0}\frac{\partial P}{\partial \sigma},\quad \alpha=\frac{TU_0}{h},\quad \beta_0=\frac{Tn}{h} \tag{16}$$

Let's look for the solution of Eq.(12) in case of boundary conditions (13 to 15) in the form of a sum [28]:

$$U(\sigma,x,\tau)=U_1(x,\tau)+U_2(\sigma,x,\tau), \tag{17}$$

where $U_1(x,\tau)$ is

$$\frac{\partial U_1}{\partial \tau}=-F(\tau)-\beta_0\left(\frac{\partial U_1}{\partial x}+x\frac{\partial^2 U_1}{\partial x^2}\right) \tag{18}$$

the solution of the inhomogeneous differential Eqs. (13 to 15) under the boundary conditions, and $U_2(\sigma,x,\tau)$

$$\frac{\partial U_2}{\partial \tau}+\alpha\frac{\partial U_2}{\partial \sigma}=-\beta_0\left(\frac{\partial U_2}{\partial x}+x\frac{\partial^2 U_2}{\partial x^2}\right) \tag{19}$$

the solution of a homogeneous differential Eq. (19) in case of zero boundary conditions.
Let's look for the solution of Eq. (19) in the form of a sum [28]:

$$U_2(\sigma,x,\tau)=\sum_{k=1}^{\infty}C_k(\sigma,\tau)J_1\left(\lambda_k\sqrt{x}\right). \tag{20}$$

From the boundary condition (15) it follows that $\frac{\partial U}{\partial x}\Big|_{x=1}=\frac{\partial U_1}{\partial x}\Big|_{x=1}+\frac{\partial U_2}{\partial x}\Big|_{x=1}=0$, therefore $\frac{\partial U_1}{\partial x}\Big|_{x=1}=\frac{\partial U_2}{\partial x}\Big|_{x=1}=0$ from which we get $J_1'(\lambda_k)=0.$ This Eq. will be the characteristic Eq. of the problem, and its roots $\lambda_k>0$ will be the eigenvalues of the problem. In order to determine the values of the coefficient C_k, substituting Eq.(20) into Eq.(19) and we get

$$\sum_{k=1}^{\infty}\left\{\frac{\partial C_k}{\partial \tau}+\alpha\frac{\partial C_k}{\partial \sigma}\right\}J_1(\lambda_k x)=\frac{1}{4}\beta_0\sum C_k\left(1-\frac{1}{\lambda_k^2 x}\right)J_1\left(\lambda_k\sqrt{x}\right) \tag{21}$$

Multiplying both parts of the resulting Eq.(20) by the expression $J_1(\lambda_m x)dx$ and integrating in the interval [0,1] and taking into consideration the orthogonality condition of the $J_1(\lambda_k x)$ function

$$\int_o^1 J_1\left(\lambda_k\sqrt{x}\right)\cdot J_1\left(\lambda_m\sqrt{x}\right)dx=\begin{cases}0 & \text{when } k\neq m\\ \int_0^1 J_1^2\left(\lambda_k\sqrt{x}\right)dx=\dfrac{\lambda_k^2-1}{\lambda_k^2}, & \text{when } k=m\end{cases} \tag{22}$$

we will have

$$\frac{\lambda_k^2-1}{\lambda_k^2}J_1^2(\lambda_k)\left\{\frac{\partial C_k}{\partial \tau}+\alpha\frac{\partial C_k}{\partial \sigma}\right\}=\frac{\beta_0}{4}L_2(\lambda_k)C_k, \tag{23}$$

where

$$L_2(\lambda_k)=\int_0^1\left(1-\frac{1}{\lambda_k^2 x}\right)J_1^2\left(\lambda_k\sqrt{x}\right)dx=\frac{1}{\lambda_k^4}\left[\left(\lambda_k^4+1\right)J_1^2(\lambda_k)-\lambda_k^2\right] \tag{24}$$

Taking into account Eq. (24) we get

$$\frac{\partial C_k}{\partial \tau}+\alpha\frac{\partial C_k}{\partial \sigma}=\beta\cdot\beta_k\cdot C_k \tag{25}$$

where

$$\beta=\frac{\beta_0}{4},\qquad \beta_k=\frac{\left[\left(\lambda_k^2-1\right)J_1^2(\lambda_k)-\lambda_k^2\right]}{\lambda_k^2\left(\lambda_k^2-1\right)J_1^2(\lambda_k)} \tag{26}$$

Let's look for the solution of Eq. (25) in the following form

$$C_k(\sigma,\tau)=D_k(\tau)\exp\left(-\lambda_k^2\sigma\right) \tag{27}$$

We will have

$\frac{dD_k(\tau)}{d\tau} = \left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)D_k(\tau)$, from which

$$D_k(\tau) = C_{0k}\exp\left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)\tau. \qquad (28)$$

Substituting $F_k(\tau)$ function value in Eq.(27) we get

$$C_k(\sigma,\tau) = C_{0k}\exp\left(\left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)\tau\right)\exp\left(-\lambda_k^2\sigma\right). \qquad (29)$$

Substituting $C_k(\sigma,\tau)$ function value in Eq.(20) we get

$$U_2(\sigma,x,\tau) = \sum_{k=1}^{\infty} C_{0k}\exp\left(\left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)\tau\right)\exp\left(-\lambda_k^2\sigma\right)J_1\left(\lambda_k\sqrt{x}\right). \qquad (30)$$

the value of the constant C_{0k} is determined from condition (14) of the problem

$$U(0,x,0) = \phi(x) = \sum_{k=1}^{\infty} C_{0k}J_1\left(\lambda_k\sqrt{x}\right). \qquad (31)$$

Let us multiply both parts of the Eq.(30) by $J_1\left(\lambda_m\sqrt{x}\right)dx$ and integrate in the interval [0,1]. Taking into account the condition of orthogonality of functions (22), we have

$$C_{0k} = \frac{\lambda_k^2 L_1(\lambda_k)}{\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)}, \qquad (32)$$

where

$$L_1(\lambda_k) = \int_0^1 \phi(x)J_1\left(\lambda_k\sqrt{x}\right)dx. \qquad (33)$$

Substituting C_{0k} value from Eq.(32) into Eq.(30) we get

$$U_2(\sigma,x,\tau) = \sum_{k=1}^{\infty}\frac{\lambda_k^2 L_1(\lambda_k)}{\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)}\exp\left(\left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)\tau\right)\exp\left(-\lambda_k^2\sigma\right)J_1\left(\lambda_k\sqrt{x}\right). \qquad (34)$$

To determine the function $U(\sigma,x,\tau)$, it is necessary to determine the also function $U_1(\sigma,x,\tau)$, which is determined by Eq. (18). Let's look for the function $U_1(\sigma,x,\tau)$ in the following form

$$U_1(x,\tau) = \sum_{k=1}^{\infty} B_k\exp\left(-\lambda_k^2\tau\right)J_1\left(\lambda_k\sqrt{x}\right) \qquad (35)$$

to satisfy Eq. (18). Substituting Eq. (35) into Eq.(18) we get

$$-\sum_{k=1}^{\infty}\lambda_k^2 B_k\exp\left(-\lambda_k^2\tau\right)J_1\left(\lambda_k\sqrt{x}\right) = -F(\tau) + \beta\sum_{k=1}^{\infty} B_k\left(1 - \frac{1}{\lambda_k^2 x}\right)\exp\left(-\lambda_k^2\tau\right)J_1\left(\lambda_k\sqrt{x}\right) \qquad (36)$$

Let's analyze the function $F(\sigma,\tau)$ in a series of eigenfunctions

$$F(\tau) = \sum_{k=1}^{\infty} a_k J_1\left(\lambda_k\sqrt{x}\right) \qquad (37)$$

where

$$a_k = \frac{\lambda_k^2 F(\tau)L_0}{\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)}, \quad L_0(\lambda_k) = \int_0^1 J_1\left(\lambda_k\sqrt{x}\right)dx. \qquad (38)$$

In case $F(\sigma,\tau) = Const = C_0$ we have

$$a_k = \frac{\lambda_k^2 C_0 L_0}{\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)}. \qquad (39)$$

From Eqs. (35 to 39) we determine values of coefficient B_k

$$B_k = \frac{a_k}{\lambda_k^2 + \beta\beta_k} = \frac{\lambda_k^2 C_0 L_0}{\left(\lambda_k^2 + \beta\beta_k\right)\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)}. \tag{40}$$

Taking into account the values of the coefficients B_k, the function $U_1(x,\tau)$ will look like this

$$U_1(x,\tau) = \sum_{k=1}^{\infty} \frac{\lambda_k^2 C_0 L_0}{\left(\lambda_k^2 + \beta\beta_k\right)\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)} \exp\left(-\lambda_k^2 \tau\right) J_1\left(\lambda_k \sqrt{x}\right) \tag{41}$$

From Eqs. (41,34, and 17) the general solution of the problem

$$U(\sigma,x,\tau) = \sum_{k=1}^{\infty} \frac{\lambda_k^2 C_0 L_0}{\left(\lambda_k^2 + \beta\beta_k\right)\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)} \exp\left(-\lambda_k^2 \tau\right) J_1\left(\lambda_k \sqrt{x}\right) + \sum_{k=1}^{\infty} \frac{\lambda_k^2 L_1(\lambda_k)}{\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)} \exp\left(\left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)\tau\right) \exp\left(-\lambda_k^2 \sigma\right) J_1\left(\lambda_k \sqrt{x}\right) \tag{42}$$

The resulting solutions refer to the general boundary and initial conditions of the problem.

5. Results of the study of structural changes in hydrodynamic parameters of unsteady turbulent plane-parallel pressure flow in the entrance region

5.1. Results of the hydrodynamics entrance region structural changes' studies in case of plane-parallel pressure unsteady turbulent flow.

Using general solutions, we can get solutions adequate to the given conditions for each individual case. Consider two private cases.

1. The velocity of the fluid entering the pipe is constant, due to which $\phi(x) = A_0 = Const, \quad L_1 = A_0 L_0$. Therefore

$$U(\sigma,x,\tau) = \sum_{k=1}^{\infty} \left[A_0 \exp\left(\left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)\tau\right) \exp\left(-\lambda_k^2 \sigma\right) + \frac{C_0 \exp\left(-\lambda_k^2 \tau\right)}{\left(\lambda_k^2 + \beta\beta_k\right)} \right] \frac{\lambda_k^2 L_0 J_1\left(\lambda_k \sqrt{x}\right)}{\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)} \tag{43}$$

2. The velocity of the fluid entering the pipe varies according to the law of the quadratic parabola, due to which $\phi(x) = A_0 x^2, \quad L_1 = A_0 \int_0^1 x^2 J_1\left(\lambda_k \sqrt{x}\right) dx = A_0 L_3,$

where $L_3 = \int_0^1 x^2 J_1\left(\lambda_k \sqrt{x}\right) dx.$

Therefore

$$U(\sigma,x,\tau) = \sum_{k=1}^{\infty} \left[A_0 L_3 \exp\left(\left(\alpha\lambda_k^2 + \beta \cdot \beta_k\right)\tau\right) \exp\left(-\lambda_k^2 \sigma\right) + \frac{C_0 \exp\left(-\lambda_k^2 \tau\right)}{\left(\lambda_k^2 + \beta\beta_k\right)} \right] \frac{\lambda_k^2 L_0 J_1\left(\lambda_k \sqrt{x}\right)}{\left(\lambda_k^2 - 1\right)J_1^2(\lambda_k)} \tag{44}$$

Regularities (43), (44) of changes in axial velocities in the inlet region of plane-parallel unsteady turbulent pressure motion were obtained. Using the identified formulas, numerical calculations were performed and velocity profiles were plotted and the length hydrodynamic entrance region was determined.

5.2. Plots of changes in the unsteady turbulent flow's hydrodynamic parameters in the hydrodynamics entrance region

5.2.1. Graphs of structural changes in parameters in the first case

Based on the results obtained, the nature of the flow features in the inlet section of plane-parallel unsteady turbulent flow was studied. To visualize regularities of development of the velocity profile along the cross section and along the length of the transition section, depending on the distribution of velocities in the inlet section, numerical calculations were carried out and graphs of their changes were constructed. Figures 1 to 4 show cases when the entering fluid velocity is constant, that is $\phi(x) = A_0 = Const$.

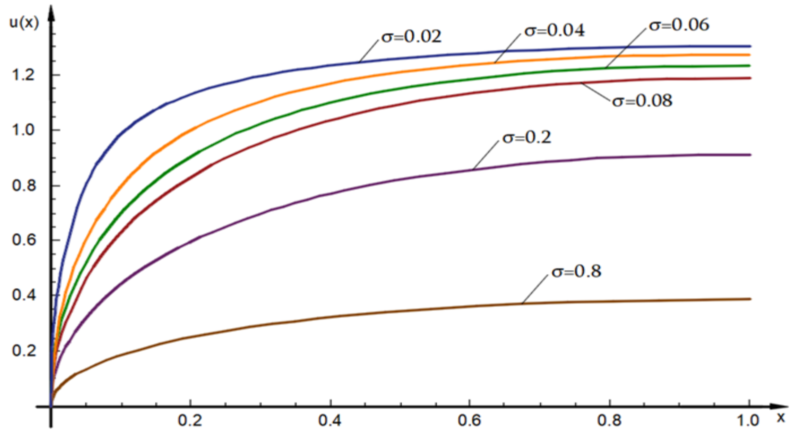

Figure 1: Axial velocity change along the hydrodynamic entrance region in case of plane-parallel unsteady turbulent motion at $A_0 = 1$, C=5, $\alpha = 0.01, \beta = 0.1, \tau = 0.5$ $\sigma \to \{0.02,0.04,0.06,0.08,0.2,0.8\}$

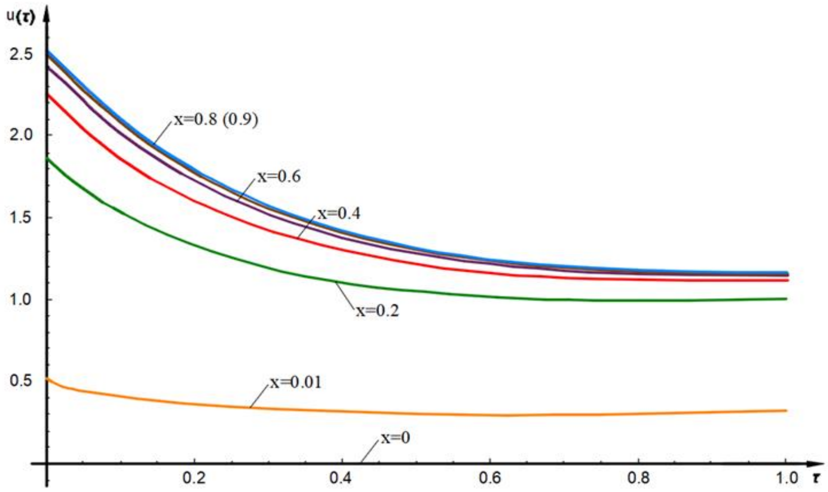

Figure 2: Axial velocity time-dependent-change at A_0=1; C=5;$\alpha = 0.01, \beta = 0.1, \sigma \to \{0.04\}$ in the interval $\{\tau, 0; 1\}$

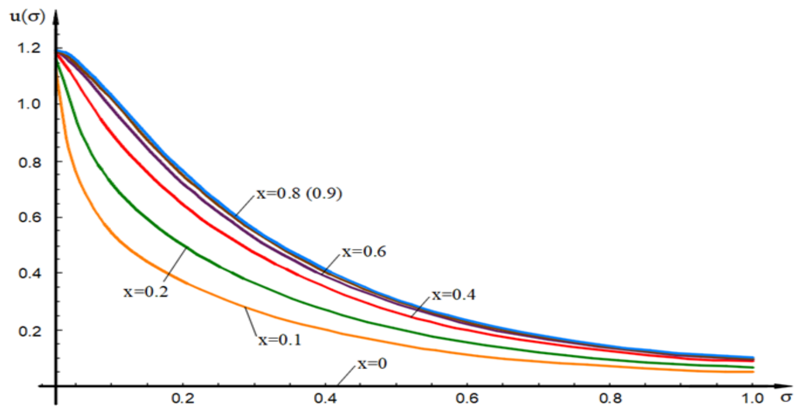

Figure 3: Axial velocity change at various points of the inlet section of the hydrodynamic entrance region at $= 1, C = 5, \alpha = 0.01, \beta = 0.1$ $\{\sigma, 0.02, 1\}, \tau \to \{1\}\}$

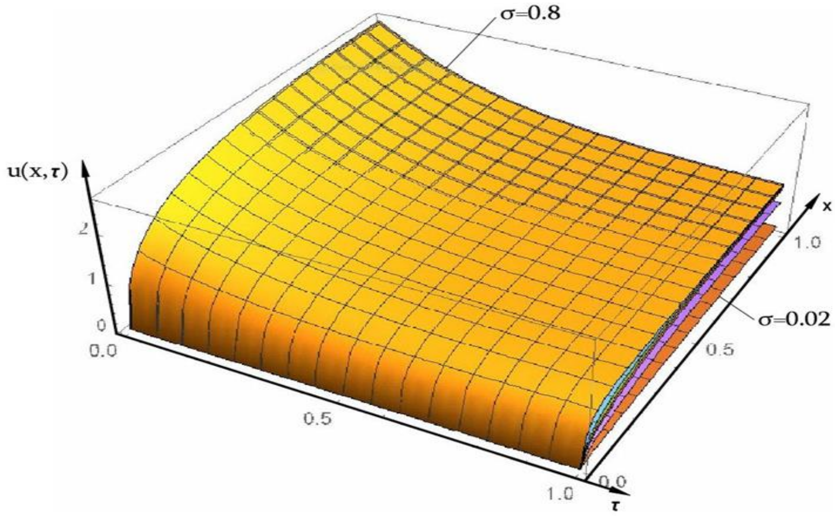

Figure 4: 3D view of the of axial velocity change in the hydrodynamic entrance region in case of plane parallel flow $\{\tau, 0; 1\}, \{x, 0; 1\}, \sigma \to \{0.02, 0.04, 0.06, 0.08, 0.2, 0.8\}$

5. 2. 2. Graphs of structural changes in parameters in the second case

Plots showing changes in the structural characteristics of an unstable turbulent flow with a parabolic law of velocity distribution at the inlet section of plane-parallel pressure motion, i.e. $\phi(x) = A_0 x^2$

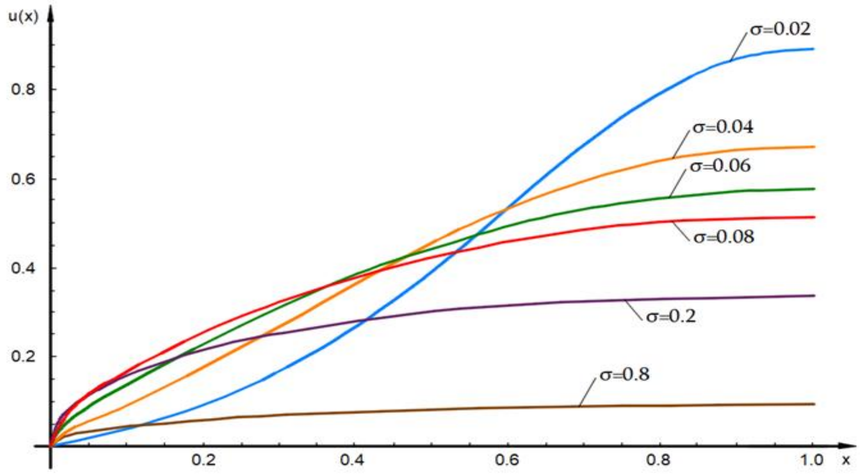

Figure 5: Axial velocity change along the hydrodynamic entrance region in case of plane-parallel unsteady turbulent motion at, $A = 1, C = 5, \alpha = 0.01, \beta = 0.1, \tau = 1, \sigma \to \{0.02, 0.04, 0.06, 0.08, 0.2, 0.8\}$

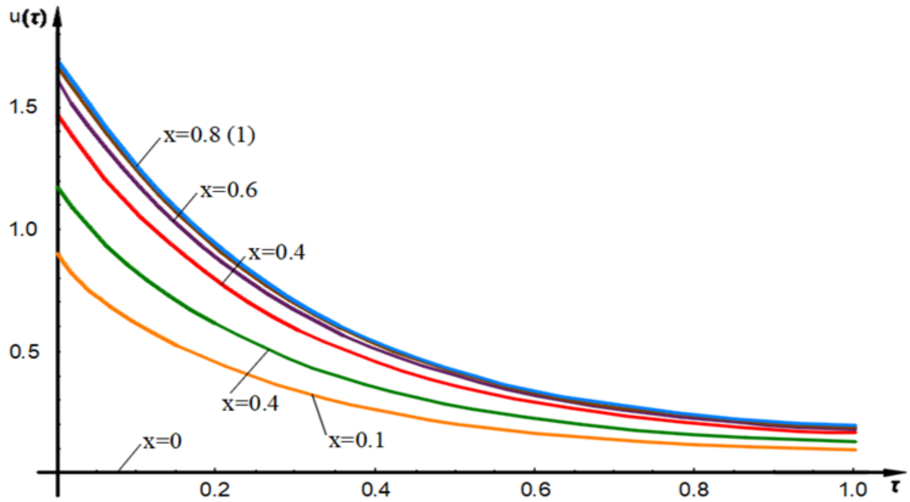

Figure 6: Axial velocity change $U(\sigma, x, \tau)$ in the entrance region of a round pipe at $\sigma \to \{0.4\}\}$, $\phi(x) = A_0 x^2$, $A = 1, C = 5, \alpha = 0.01, \beta = 0.1,$

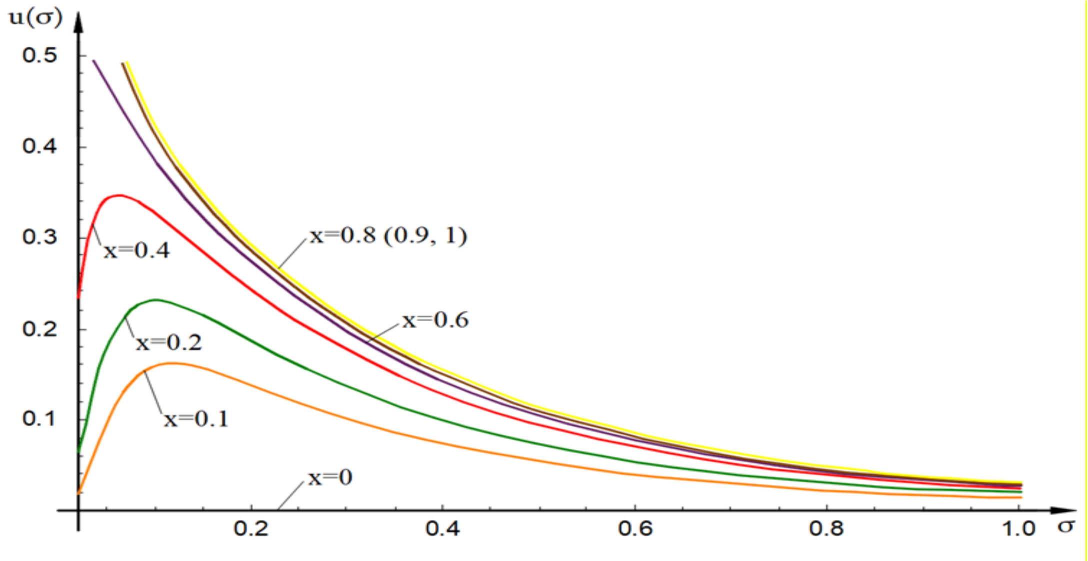

Figure 7: Axial velocity change $U(\sigma, x, \tau)$ along the length of entrance region at $\{\sigma, 0.02; 1\}$, $\tau = 1$, $\phi(x) = A_0 x^2$, $A = 1, C = 5, \alpha = 0.01, \beta = 0.1,$

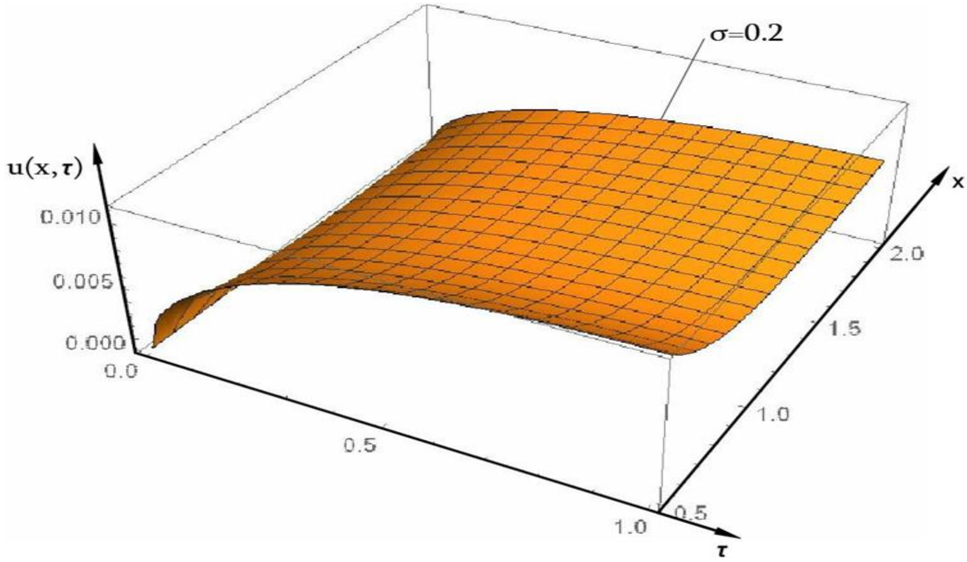

Figure 8: 3D view of the of axial velocity change in the hydrodynamic entrance region in case of plane parallel flow at, $\phi(x) = A_0 x^2$, $\{x, 0,1\}, \{\tau, 0.5,2\}$

6. Discussion

Based on the dependency of the tangential stresses between the fluid layers on the distance of the stationary interior wall of the pipe, the unsteady plane-parallel turbulent flow of an incompressible fluid in the hydrodynamic entry zone was studied. An integration of the boundary value issue was performed in order to get the flow parameters. Consequently, a velocity profile with arbitrary boundary conditions was projected over the transition region's length. Two particular situations were examined based on the general solutions found: the first had a constant entering fluid velocity, and the second involved a fluid velocity change that followed a parabolic equation.

Plots of the process at the hydrodynamic entrance region of the flat pipe have been created by computer calculations utilizing formulae (42) and (43) to indicate the evolution of the process. The pressure gradient and the starting velocity distribution in the intake section determine how much the process develops over time, according to a study of the numerical calculation results and the ensuing graphs (Figs. 1 to 8).

During unsteady flow, the viscous fluid flow in the hydrodynamic entrance region is unstable. The pressure gradient is what causes the change in velocity profile outside of the hydrodynamic entrance region. According to the theories, the evolution of a viscous fluid's flow in a flat pipe's entrance point was studied taking account turbulent shear forces. In engineering practice, the accuracy of integration findings is quite acceptable. The study's findings may be used to the design of the hydraulic systems' input units for a variety of mechanisms and equipment, providing more reliable and smooth operation.

Given the problem's importance, its thorough examination is necessary to determine the patterns of hydrodynamic parameters of an unstable turbulent flow at the plane-parallel motion inlet as well as to explain the dependency of the kinematic viscosity coefficient.

Analysis of the results of numerical calculations and the resulting graphs determined the length of the initial section. The deviation of the axial velocity in the transition section at x=1 should not exceed 1% of the unsteady speed of the stabilized section. Based on this condition, a calculation formula was obtained for determining the length of the transition section, which has important practical applications in the design of various hydraulic automation systems.

7. Conclusions

1. The proposed suitable method of calculation of velocity rearrangement in the hydrodynamic entrance region of the plane parallel motion makes it possible to obtain the regularities of changes in the hydrodynamic parameters of the flow under general boundary conditions. In order to achieve this paper's goal, the differential equation of the motion of an elementary jet of a real liquid was drawn up, in which the frictional stresses arising between the layers were considered depending on the distance from the stationary wall. The boundary conditions of the problem were stated. A methodology of solving the defined boundary problem was developed, as a result of which regularities of axial velocity change were obtained at the hydrodynamic entrance region, under conditions of constant and parabolic law distribution of the incoming fluid.
2. Using the identified relations, computer analyzes were carried out, as a result of which the course of the deformation of the velocity profile in the transition region was obtained. It makes it possible to calculate the changes in the hydrodynamic parameters of the flow and make generalizations. Findings of regularities of velocity profile changes in the hydrodynamic entrance region and plots based on them make it possible to correctly design reliable functioning units of hydro-mechanical equipment.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

Manuscript has no associated data

Use of artificial intelligence

The authors have used artificial intelligence technologies within acceptable limits to provide their own verified data, which is described in the research methodology section.

References

1. Ainola L.Ya., Ruustal (1985). Flow development at the inlet section of a round pipe during accelerating fluid movement. Proceedings of the Tallinn Polytechnic Institute, Tallinn, pp 95-107,
2. A. Reci, A. Sederman, L. Gladden (2018) Experimental evidence of velocity profile inversion in developing laminar flow using magnetic resonance velocimetry. Journal of Fluid Mechanic, DOI:10.1017/jfm.2018.512,
3. K. Urbanowicz, M. Firkowski, A Bergant. (2018) Comparing analytical solutions for unsteady laminar pipe flow. Conference. BHR Pressure surges.
4. T. Wiens, E. Etminan An Analytical Solution for Unsteady Laminar Flow in Tubes with a Tapered Wall Thickness, *Fluids* 2021, *6*(5), 170; https://doi.org/10.3390/fluids6050170
5. A. E. Vardy, M. B. Brown. (2011) Laminar pipe flow with time-dependent viscosity, Journal of Hydroinformatics 13 (4): 729–740. https://doi.org/10.2166/hydro.2010.073,
6. Sarukhanyan A., Vartanyan Y., Baljyan P., Vermishyan G. (2023). Pattern identification of the non-stationary laminar flow of a viscous fluid in the round pipe inlet section. Eastern-european journal of enterprise technologies, 2(7(122)), pp.33-42; doi: 10.15587/1729-4061.
7. Sarukhanyan A., Vermishyan G., Kelejyan H. (2023). Plane-parallel laminar flow of viscous fluid in the transition zone of the inlet section. Journal of architectural and engineering research (JAER), vol.4, pp.75-85. Issn: 2738-2656, doi: https://doi.org/10.54338/27382656-2023.4-008.
8. Sarukhanyan A., Vartanyan A., Vermishyan G., Tokmajyan V. (2020). The study of hydrohydrodynamic processes occurring on transition of sudden expanding of hydraulic section of plane – parallel full pipe flow. TEM journal,volume 9, issue 4, pages 1494-1501, issn 2217-8309, doi: 10.18421/tem94-23.
9. Ashok Kannaiyan A., Natarajan S. Vinoth B. R. (2022), Stability of a laminar pipe flow subjected to a step-like increase in the flow rate. Physics of Fluids 34, 064102: https://doi.org/10.1063/5.0090337
10. B. Lebon, J. Peixinho,S.Ishizaka and Y. Tasaka. (2018) Subcritical transition to turbulence in a sudden circular pipe expansion, Journal of Fluid Mechanics , Volume 849 , pp. 340 – 354, DOI: https://doi.org/10.1017/jfm.2018.421
11. G. Shajari, M. Abbasi and M. Jamei. (2021) Entrance length of oscillatory flows in parallel plate microchannels. Journal of Mechanical Engineering Science, Volume 235 Issue 19, https://doi.org/10.1177/0954406220968125
12. M. K. Wong, L. C. Sheng, C. S. Nor Azwadi, G. A. Hashim Numerical Study of Turbulent Flow in Pipe with Sudden Expansion. Journal of Advanced Research in Fluid Mechanics and Thermal Sciences| Vol. 6, No.1. Pages 34-48, 2015 ISSN (online): 2289-7879
13. Kühnen, J., Song, B., Scarselli, D., Budanur, N.B., Riedl, M., Willis, A.P., Avila, M., and Hof, B., Destabilizing turbulence in pipe flow, *Nat. Phys*., 2018, vol. 14, pp. 386–390. https://doi.org/10.1038/s41567-017-0018-3
14. Daprà I. , Scarpi G. (2017) Unsteady Flow of Fluids With Arbitrarily Time-Dependent Rheological Behavior., Journal of Fluids Engineering 139(5), DOI:10.1115/1.4035637
15. B. Lebon, J. Peixinho,S.Ishizaka and Y. Tasaka. (2018) Subcritical transition to turbulence in a sudden circular pipe expansion, Journal of Fluid Mechanics , Volume 849 , pp. 340 – 354, DOI: https://doi.org/10.1017/jfm.2018.421
16. G. Shajari, M. Abbasi and M. Jamei. (2021) Entrance length of oscillatory flows in parallel plate microchannels. Journal

of Mechanical Engineering Science, Volume 235 Issue 19, https://doi.org/10.1177/0954406220968125

17. M. K. Wong, L. C. Sheng, C. S. Nor Azwadi, G. A. Hashim Numerical Study of Turbulent Flow in Pipe with Sudden Expansion. Journal of Advanced Research in Fluid Mechanics and Thermal Sciences| Vol. 6, No.1. Pages 34-48, 2015 ISSN (online): 2289-7879.
18. Gerardo R.,Robert J.P., Paulo J.O. Bifurcation phenomena in viscoelastic flows through a symmetric 1:4 expansion.- 2007, Journal of Non-Newtonian Fluid Mechanics 141(1):pp.1-17, DOI: 10.1016/j.jnnfm.2006.08.008.
19. Gücüyen E., Erdem R.T., , Gökkuş Ü. Numerical Modelling of Sudden contraction in pipe Flow.- 2019, Sigma J Eng & Nat Sci Vol. 37 (3)- pp.903-916.
20. Mullin T., Seddon J. R. T., Mantle M. D., *and* Sederman A. J. Bifurcation phenomena in the flow through a sudden expansion in a circular pipe.- 2009, Physics of Fluids 21, 014110; https://doi.org/10.1063/1.3065482.
21. Takumi H., Zvi R. Viscous flow in a slightly asymmetric channel with a sudden expansion. – 2000, Physics of Fluids-12(9):pp.2257-2267, DOI: 10.1063/1.1287610.
22. Fester V., Mbiya B., Slatter P. Energy losses of non-Newtonian fluids in sudden pipe contractions.- 2008.- Chemical Engineering Journal, Yol.145(1), pp.57-63. DOI: 10.1016/j.cej.2008.03.003.
23. Chen, F. Hussain and Z. S. She. Quantifying wall turbulence via a symmetry approach. Part 2. Reynolds stresses. J. Fluid Mech. 850, 401-438 (2018).
24. R. Baidya, J. Philip,N. Hutchins,J. P. Monty,and I. Marusic Distance-from-the-wall scaling of turbulent motions in wall-bounded flows Physics of Fluids 29, 020712 (2017), https://doi.org/10.1063/1.4974354.
25. P. Luchini.Universality of the Turbulent Velocity Profile,Phys. Rev. Lett. 118, 224501 – Published 1 June 2017, DOI: https://doi.org/10.1103/PhysRevLett.118.224501.
26. Schlichting H., Gersten K., 2017, Boundary-Layer Theory. *Berlin: Springer*-Verlag *Berlin* Heidelberg. Available at: https://link.springer.com/book/10.1007/978-3-662-52919-5
27. Targ S.M. [1951]. Main Problems of the Theory of lamiar flows. Moscow. Gostekhizdat, pp 420.
28. Tikhonov A.N., Samarski A.G. [1999]. Eq.s of Mathematical Physics, Moscow. Nauka, p. 799.