

Mathemundus As A Space for Student Research and Professional Development

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Abstract

University learning workshops are gaining importance in professionally oriented teacher education because they enable students to connect theoretical concepts with practical curriculum and lesson development and scholarly reflection. Using the university learning workshop Mathemundus as an example, this paper examines how pre-service teachers develop mathematical learning environments for primary education, trial them in school contexts, and reflect on their experiences through research-based learning. Drawing on concepts of discovery and research-based learning, Mathemundus is conceptualised as a space for learning, development, research and transfer. The analysis shows how material development, classroom trialling, observation and theory-informed reflection can be combined in recursive processes of professional development. A standardised accompanying survey provides initial empirical indications that students experience these interconnected processes as relevant to their future teaching practice and professional judgement. The paper develops the Workshop of Teaching as a higher-education model of recursive professionalisation. Research-based learning is understood not merely as a teaching method but as a professional disposition that enables pre-service teachers to view teaching as a process that can be analysed, justified and continuously developed. The paper concludes by discussing the potential and limitations of workshop-based teacher education and outlines perspectives for further empirical investigation.

Keywords: recursive professionalization; university learning workshops; professional judgement; research-based learning; teacher education; higher education didactics; workshop of teaching; mathematics education.

1. Introduction

Professional pedagogical practice does not arise solely from the acquisition of subject-specific and subject-didactic knowledge. It develops through the systematic interrelation of scholarly reflection, practical trialling and critical analysis of concrete teaching and learning processes [1-3]. Teacher education therefore faces the challenge of enabling pre-service teachers not only to receive research-based knowledge but also to engage independently and reflectively with teaching as an object of inquiry. University learning workshops offer particularly favourable conditions for this purpose because they create spaces in which students develop, trial, analyse and refine teaching on the basis of theoretical concepts.

In recent years, university learning workshops have become established as distinct higher-education formats that connect research-based learning, student agency and reflective professionalisation. They create opportunities for students to formulate their own questions, justify didactic decisions and experience teaching as an open developmental process [4-7]. While much of the literature focuses on theoretical foundations or descriptions of individual workshop formats, less attention has been paid to how workshop-based learning contributes concretely to research-oriented professionalisation and which higher-education structures support these processes.

Against this background, the present paper examines the university learning workshop Mathemundus as a space for student research and professional development. The central question is how pre-service teachers develop mathematical learning environments, trial them in school contexts and reflect on their experiences through theoretical perspectives. Mathemundus is understood not only as a place of learning but as a space for development, research and transfer in which theory, practice and scholarly reflection are recursively related.

The paper elaborates the contribution of workshop-based higher education to the professionalisation of pre-service teachers and uses Mathemundus as the starting point for a higher-education model of professionalisation. It first outlines theoretical perspectives on professionalisation, university learning workshops, discovery learning and research-based learning. It then presents the conception of Mathemundus and analyses the interrelation of student material development, classroom trialling and scholarly reflection. Finally, the resulting professionalisation processes are discussed and perspectives for the further development of workshop-oriented teacher education are outlined.

With the Workshop of Teaching, the paper does not propose another learning-workshop format; rather, it develops a theoretical perspective on processes of professionalization in higher education.

2. Professionalization as a Guiding Perspective in Teacher Education

Professionalisation is a central objective of teacher education. It encompasses considerably more than the acquisition of subject knowledge, subject didactics and educational knowledge. Professional teaching develops through the capacity to analyse classroom situations theoretically, make justified pedagogical decisions and continuously reflect on and develop one's own practice [1,2,8]. Teacher education must therefore provide learning opportunities in which students experience teaching not only as an object of didactic planning but also as an object of professional judgement.

This paper understands professionalisation as a recursive developmental process. Professional competence does not develop linearly through the acquisition of academic knowledge followed by its application in practice. Rather, it emerges through a recurring interplay of theoretical grounding, practical trialling, systematic observation, scholarly reflection and conceptual refinement.

This understanding connects with approaches that conceptualise professional development as reflective and research-oriented. Professional practice emerges when students do not merely manage pedagogical situations but make them objects of scholarly analysis. Reflection has a dual function: it supports critical engagement with one's own actions and enables theoretical concepts to be reconsidered in the light of concrete classroom experience [3,9].

This perspective also changes the view of higher-education learning arrangements. What matters is not only which content is taught but under which conditions students can develop professional judgement. Learning environments gain quality when they enable autonomous developmental processes, scholarly reflection and engagement with authentic classroom situations [9]. The often-observed distance between knowledge and action in professional learning further underlines the need for formats that systematically connect these domains [21].

University learning workshops are particularly relevant from this perspective. They provide spaces in which research findings are not merely received but actively taken up, trialled and developed further. Students assume responsibility for designing mathematical learning environments, observe their implementation, analyse learning processes and revise their concepts on the basis of theoretical and empirical insights [5,6,10]. The workshop thus becomes a place in which professional judgement develops in the tension between theory, practice and scholarly reflection.

This understanding forms the point of departure for the present paper. Its guiding assumption is that professionalisation is particularly likely to occur when teaching itself becomes an object of scholarly development. The following sections reconstruct how Mathemundus can provide conditions for such recursive processes.

A recursive understanding of professionalisation also requires a distinction between reflection as retrospective description and reflection as a form of professional inquiry. If reflection remains limited to whether a lesson 'worked', it risks reproducing intuitive judgements without making their underlying assumptions visible. In the present model, reflection is therefore tied to theoretical concepts, observable learning processes and

the explicit justification of didactic decisions. This connection is important because professional judgement is not treated as an individual disposition detached from knowledge, but as a capacity that develops when different forms of knowledge are brought into relation with situated experience [1-3].

This perspective is especially relevant in teacher education because the relationship between academic knowledge and classroom action cannot be reduced to simple transfer. The distance between knowing and acting described in professional learning research [21] suggests that opportunities are needed in which students repeatedly translate theoretical ideas into design decisions, encounter the consequences of these decisions in authentic situations and return to theory with new questions. Recursive professionalisation names precisely this movement. It does not imply repetition of the same activity; rather, each cycle can modify the questions, criteria and interpretive frames with which subsequent practice is approached.

3. University Learning Workshops as Spaces for Research-Oriented Professionalisation

Against this professional-theoretical background, university learning workshops gain particular significance. They combine research knowledge with students' own processes of development, trialling and reflection and thereby create opportunities for research-oriented professional learning [5,6].

University learning workshops differ from strongly instructional formats and connect with constructivist and research-oriented approaches to higher education [4,9]. Rather than separating the transmission of academic content from its application in school practice, they combine scholarly thinking, didactic development and practical trialling in recursive learning processes. Students become designers of learning environments, observers of learning processes and reflective analysts of their own pedagogical actions.

In learning-workshop research, such workshops are therefore understood not primarily as special rooms or collections of materials but as higher-education concepts that enable autonomous learning, openness and scholarly reflection [5,7]. The broader discourse also emphasises that materiality, interaction, experience and reflection interact within these spaces [11].

Holub and Roszner [10] extend this perspective by showing how university learning workshops can be curricularly embedded places in which academic, didactic and practical experiences are systematically connected. Particularly relevant is the pedagogical double-decker: students first experience workshop-based learning from the learner perspective, reflect on this experience and subsequently transfer it to the design of their own teaching and learning processes. This recursive relation between experience, theory and action corresponds with reflective approaches to professional learning [3,9].

Research-based learning is a central structural principle in this context. Students engage with their own questions using scholarly modes of thinking and working and thereby develop reflective competences that are central to professional pedagogical action [4,9]. In the context of learning workshops, research-based learning is not limited to carrying out formal research projects; it also denotes a professional disposition characterised by openness, systematic observation, justified analysis and willingness to develop one's own practice.

University learning workshops thus make it possible to experience teaching as a field of research and development. Material development, classroom trialling, documentation and reflection are not separate steps but interrelated elements of professional learning. Earlier work on workshop-based teacher education likewise highlights openness, teacher research and the productive linking of theory and practice [17-20].

Mathemundus is situated within this perspective. It is not merely a collection of mathematics education materials or a place for hands-on learning. It is conceived as a higher-education setting in which pre-service teachers develop mathematical learning environments on a theoretical basis, trial them in school contexts, systematically observe learning processes and use the resulting insights to refine teaching.

The following sections therefore consider Mathemundus as a model of workshop-based higher education in which research-based learning is systematically linked with processes of professionalisation.

Openness is a further defining feature of workshop-based learning. It does not mean the absence of structure, but the deliberate creation of spaces in which different approaches, representations and solutions can become visible. This is particularly significant in mathematics education, where the didactic quality of a task is often revealed through the diversity of learners' strategies rather than through the reproduction of a prescribed procedure. Earlier work on openness and research-oriented mathematics education has shown that such settings can create productive starting points for inquiry when learners are encouraged to articulate, compare and justify their approaches [17-20]. For pre-service teachers, this means that openness becomes both an object of didactic planning and an experience that challenges them to respond to unexpected learning processes.

University learning workshops can therefore function as boundary spaces between academic study and school practice. Their contribution is not that they dissolve the differences between these domains, but that they make these differences pedagogically productive. Academic concepts provide categories for interpretation; school-based experiences confront students with the contingency and complexity of real learning processes; collaborative reflection creates opportunities to relate both perspectives without reducing one to the other. In this sense, the workshop offers a structured setting in which uncertainty is not treated as a deficit to be eliminated immediately, but as a starting point for professional questioning and further inquiry [5-7,10].

4. Discovery and Research-Based Learning as Foundations of Recursive Professionalisation

The development of professional judgement requires learning opportunities in which students do not merely receive knowledge but construct, test and develop it independently. Discovery learning and research-based learning therefore form the didactic basis of the understanding of professionalisation developed in this paper. Both approaches assume that sustainable learning emerges when learners actively formulate questions, develop hypotheses, reflect on experiences and transform insights into new perspectives for action [12].

Discovery learning conceptualises learning as an active process of construction. Knowledge is built through engagement with problems, materials and situations. Bruner [12] describes learning as a process of discovery in which relationships are independently recognised. Winter [13,14] transfers this idea to mathematics education and emphasises that mathematical insights become robust when learners discover structures themselves, justify them and relate them to one another. Liebig [15,16] describes discovery learning as a teaching principle that deliberately supports agency, openness and individual ways of thinking.

For teacher education, this entails a fundamental change of perspective. Pre-service teachers should not encounter discovery learning only as a method to be used with pupils; they should experience it themselves [12]. Through their own experiences with open learning processes, they can develop an understanding of the subject-specific, didactic and communicative demands involved in designing such learning environments.

Whereas discovery learning focuses on learners' processes of gaining insight, research-based learning adds a scholarly dimension. It aims to familiarise students with academic modes of thinking and working and to enable them to investigate pedagogical questions independently. Huber [4] describes research-based learning as a form of learning in which students engage with the research process from the development of a question through analysis to reflection on findings. Reinmann [9] emphasises the reflective competences that can emerge from such processes.

In this paper, research-based learning therefore denotes less a single method than a professional stance towards teaching; pedagogical decisions are systematically observed, theoretically justified and critically reflected upon. Teaching itself becomes an object of scholarly inquiry.

The professionalisation potential of university learning workshops lies particularly in connecting discovery and research-based learning. Discovery learning opens autonomous approaches to mathematical content and didactic questions [4,14], while research-based learning enables systematic analysis and scholarly reflection. New insights emerge recursively through development, trialling, observation, analysis and further development.

Within university learning workshops, discovery and research-based learning can thus be directly intertwined: students design learning environments, trial them with children, observe learning processes, reflect on their experiences in the light of research and revise their concepts. This interrelation creates spaces for professional judgement.

Professionalisation, from this perspective, cannot be understood as a linear sequence of university study followed by practice. It develops through recurring processes in which academic theory and pedagogical practice are continuously related. The following section examines how Mathemundus is designed to support such recursive processes.

The distinction between discovery learning and research-based learning is particularly important for the model developed here. Discovery learning primarily concerns the way in which learners gain access to subject matter through active exploration

and the recognition of relationships. Research-based learning adds a second-order perspective: experiences, observations and emerging questions become objects of systematic inquiry. In teacher education, the two perspectives can therefore be linked without being conflated. Pre-service teachers can experience discovery-oriented learning themselves, design comparable opportunities for children and subsequently examine what becomes visible in these learning processes through a research-oriented lens [4,12-16].

This double movement also explains why research-based learning is understood here as a professional disposition rather than as the isolated performance of a research method. A disposition towards inquiry becomes visible when students suspend premature judgements, attend closely to children's strategies, seek evidence for didactic assumptions and remain willing to revise their own designs. Such an orientation is compatible with reflective practice [3] but adds an explicit emphasis on systematic observation, documentation and theory-informed interpretation. The aim is not to turn every teaching situation into a formal research project. Rather, it is to cultivate a mode of professional attention in which teaching remains open to analysis and development.

5. Mathemundus – The Workshop of Teaching

The preceding considerations lead to an understanding of Mathemundus as a concrete realisation of a higher-education concept that is termed the Workshop of Teaching in this paper.

The term Workshop of Teaching refers less to a physical place than to a mode of professional work. Teaching is understood as an object of continuous development: academic knowledge, didactic design, practical trialling and reflective analysis interact in recurring developmental cycles.

Within this model, material development functions as a medium of professional engagement. By developing mathematical learning environments, students make and justify subject-specific and subject-didactic decisions, anticipate possible learning processes and test their assumptions in subsequent classroom trialling.

This understanding shapes Mathemundus. Mathematical learning environments for primary education are developed on a theoretical basis, trialled in classrooms and further developed through observation, reflection and scholarly knowledge. The focus is therefore not the finished material itself but the developmental process experienced by the students.

Mathemundus thus constitutes a multidimensional space of professionalisation in which subject learning, didactic development, observation of children's learning processes, collegial exchange and scholarly reflection interact.

The Workshop of Teaching serves as a theoretical frame for making visible how recursive professionalisation processes can be designed within teacher education.

From this perspective, the central question is not which materials are produced in the university learning workshop, but which processes of professional development are made possible through their development. The following section addresses this question through the student development and research processes in Mathemundus.

The specific contribution of Mathemundus lies in making this conceptual relation operational through mathematics-related design work. Material development creates a tangible object around which theoretical, didactic and practical questions can be negotiated. Decisions about representations, task structures, language, degrees of openness and possible forms of differentiation must be made explicit in the material and in the learning, environment surrounding it. These decisions are therefore accessible to discussion before classroom use and can later be reconsidered in relation to children's actual responses. The material functions as a provisional didactic hypothesis: it embodies assumptions about how mathematical ideas might become accessible, while classroom trialling provides occasions to examine and revise these assumptions.

This perspective also guards against a common misunderstanding of hands-on materials. Materiality alone does not guarantee discovery, understanding or professional learning. The decisive issue is how materials are embedded in questions, interactions and opportunities for reflection. In Mathemundus, the professional value of material development lies precisely in the requirement to connect the concrete design with subject-didactic reasoning. Students are asked not only what a material enables children to do, but which mathematical structures it makes visible, which strategies it may encourage, where misconceptions might arise and how observations from its use should influence further development. The workshop thus shifts attention from the attractiveness of a product to the quality of the reasoning that accompanies its design and revision.

A further dimension is collaboration. The development and analysis of learning environments take place in exchange with peers, teacher educators and school practitioners. This plurality of perspectives is significant because professional judgement is strengthened when interpretations must be articulated, compared and justified rather than remaining private impressions. Collaborative discussion can reveal different readings of the same classroom episode and can prompt students to reconsider assumptions that had previously remained implicit. The Workshop of Teaching therefore conceptualises professionalisation as both an individual and a socially mediated process of inquiry.

6. Student Research in the Workshop of Teaching

In the Workshop of Teaching, students take on the roles of designers, researchers and reflective practitioners. Their professionalisation unfolds through the independent development, theory-informed justification, practical trialling and scholarly reflection of didactic concepts.

Student work begins with the development of mathematical learning environments. The starting points are subject-specific and subject-didactic questions derived from theoretical concepts and from the analysis of children's mathematical learning processes. This first phase already requires professional decisions: Which mathematical ideas should children discover? Which representations support robust conceptions? Which linguistic, material and methodological approaches open productive learning processes? Material development thereby becomes a process of didactic analysis rather than merely a design activity.

To accompany the theoretical development of the professionalisation model, the higher-education concept was evaluated through a standardised survey. The responses show

consistently positive assessments of central dimensions of professional learning. Particularly salient were opportunities to trial didactic action in open and learner-centred settings, to develop competence-oriented tasks for heterogeneous groups and to connect different subject perspectives. Nearly all participating students also reported gaining new insights for their future teaching. Open responses referred less to individual materials or methods than to changes in professional attitudes and perspectives on teaching and learning. The evaluation therefore provides initial empirical plausibility for the assumption that recursive professionalisation is supported when scholarly reflection, didactic development and practical trialling are systematically interconnected.

The developed learning environments are subsequently trialled in primary classrooms. School practice does not serve to verify a finished product; rather, it opens a space for inquiry in which mathematical learning processes can be observed and theoretical assumptions related to concrete classroom experiences. Students attend to children's ways of thinking, unexpected solution strategies, moments of uncertainty and situations in which mathematical understanding becomes visible. Teaching thereby becomes an object of systematic observation.

These observations provide the basis for shared reflection. In dialogue with teachers, university educators and fellow students, experiences are theoretically interpreted and analysed with regard to their didactic consequences. Reflection is not an ex-post evaluation of teaching but a reconstruction of mathematical learning processes and a basis for further development of the learning environments. Theory therefore has a dual function: it structures the analysis of observations and opens new perspectives for teaching.

A defining feature of the Workshop of Teaching is that the developmental process does not end with reflection. Insights feed directly into the revision of learning environments and become the starting point for a new cycle. Planning, development, trialling, observation and reflection do not form a linear sequence but a recursive process of professional development.

Professionalisation therefore develops through the repeated movement between scholarly analysis and pedagogical practice, rather than through isolated material development or a single teaching experience.

Mathemundus provides the higher-education framework for this movement. The central outcome of workshop activity is consequently less the individual material than the development of professional judgement.

From this perspective, the Workshop of Teaching can be reconstructed as a higher-education model of professionalisation.

The standardised accompanying survey should be interpreted within this conceptual purpose. It was used to obtain an initial indication of how students perceived central dimensions of the workshop experience, not to establish causal effects of Mathemundus on professional competence. The consistently positive assessments of opportunities to work in open and learner-centred settings, to develop competence-oriented tasks for heterogeneous groups and to connect different subject

perspectives support the plausibility of the model from the participants' perspective. The open responses are especially relevant because they point beyond satisfaction with individual materials and towards changes in how students describe teaching, learning and their future professional practice.

At the same time, the evaluation has clear limits. Self-reported perceptions cannot be equated with demonstrated professional competence, and a positive retrospective assessment does not in itself show how durable professional learning is or how it transfers to later classroom practice. The evaluation is therefore treated as accompanying evidence rather than as a validation of the model. Its function in this paper is to indicate that the dimensions theoretically identified as relevant were also experienced by students as meaningful. Future studies would need to combine such self-reports with longitudinal data, analysis of student artefacts, observation of professional decision-making and, where possible, evidence from subsequent teaching practice.

7. The Workshop of Teaching as a Model of Recursive Professionalisation

The analysis of Mathemundus allows the Workshop of Teaching to be condensed into a model of recursive professionalisation. At its centre is the interplay of academic theory, didactic development, school practice and reflective analysis.

Recursive professionalisation describes a cyclical developmental process in which professional competence emerges not through linear knowledge transmission but through the repeated interrelation of theoretical grounding, didactic design, practical trialling, systematic observation and scholarly reflection. Each cycle does more than optimise didactic materials; it extends students' professional judgement. New insights arise through feedback between scholarly analysis and pedagogical action and become the starting point for further development.

Material development has a particular function within this model. Learning environments are not the ultimate goal of workshop activity but a medium of professional inquiry. Their development makes subject-specific and didactic decisions visible, justifiable and open to reflection. Classroom trialling provides insights into children's mathematical learning processes and allows students to relate theoretical assumptions to concrete experience. Materials thereby become working instruments through which professional decisions can be developed, examined and refined.

Shared reflection is equally central. Professionalisation does not occur solely through individual learning but through scholarly exchange among different perspectives. Analysing classroom observations, discussing didactic decisions and reconnecting them to theoretical concepts supports the continuous development of professional judgement. Reflection is thus an integral element of recursive development rather than a final evaluation.

The Workshop of Teaching consequently describes neither a physical learning space nor a specific method. It denotes a higher-education understanding of teacher education in which teaching itself becomes an object of scholarly development. Students learn to understand teaching not as a finished concept to be adopted but as a process that can be designed, analysed and justified.

The model reconstructed from Mathemundus also has significance beyond the individual case. It describes basic design principles for workshop-oriented teacher education and indicates conditions under which university learning workshops can support recursive professionalisation. The Workshop of Teaching can therefore serve as a theoretical frame for systematically describing and further developing different forms of workshop-based higher education.

The model brings together theory, development, trialling, observation, reflection and transfer as recursively connected processes. The accompanying evaluation provides initial indications that students perceive the associated dimensions of professionalisation as comprehensible and relevant to their professional learning. Further empirical research is needed to examine these relationships in greater depth.

The model can be read on three interconnected levels. On the first level, students engage with mathematical content and subject-didactic concepts. On the second, they transform this knowledge into designed learning environments and encounter the responses of children in authentic classroom situations. On the third, they make both the design and the observed learning processes objects of scholarly reflection. Recursive professionalisation emerges when movement between these levels is repeated and when insights from one level change activity on the others. The model therefore does not prescribe a fixed sequence of steps; it identifies a structure of relations that can be realised in different temporal and organisational forms.

Transfer is included in this structure in two senses. First, insights gained from one cycle are transferred into the revision of the same learning environment. Second, the professional ways of seeing and reasoning developed through the process may become available for new topics and teaching situations. The latter form of transfer is particularly important for teacher education: the long-term aim cannot be the mastery of one workshop material, but the capacity to approach unfamiliar teaching situations with theoretically informed questions,

attentive observation and a willingness to revise decisions. In this sense, the model locates sustainability not primarily in the durability of materials, but in the development of professional modes of inquiry.

8. Discussion

The paper has considered Mathemundus not only as an example of innovative higher education but has reconstructed from it the Workshop of Teaching as a higher-education frame. Its conceptual contribution lies in describing professionalisation through the repeated connection of theoretical grounding, didactic development, school-based trialling, observation and reflection.

This shifts the perspective on university learning workshops. Their value lies not primarily in open learning formats or the provision of materials, but in the conditions under which students can independently develop, justify, observe and refine teaching. Building on descriptions of university learning workshops as spaces of learning, experience and research, the Workshop of Teaching places professional judgement at the centre [5-7].

Material development is likewise re-accentuated. Materials are not the primary outcome of workshop activity but a medium through which didactic decisions become visible and open to reflection. Their professionalisation potential emerges particularly through the connection with trialling, observation and theory-informed revision.

The concept of recursive professionalisation refers to this repeated feedback between theory and pedagogical action. The Workshop of Teaching is therefore better understood as a professional mode of thinking and working than as a spatial arrangement.

As summarised in Figure 1, professional judgement develops in this model through recursively connected developmental processes.

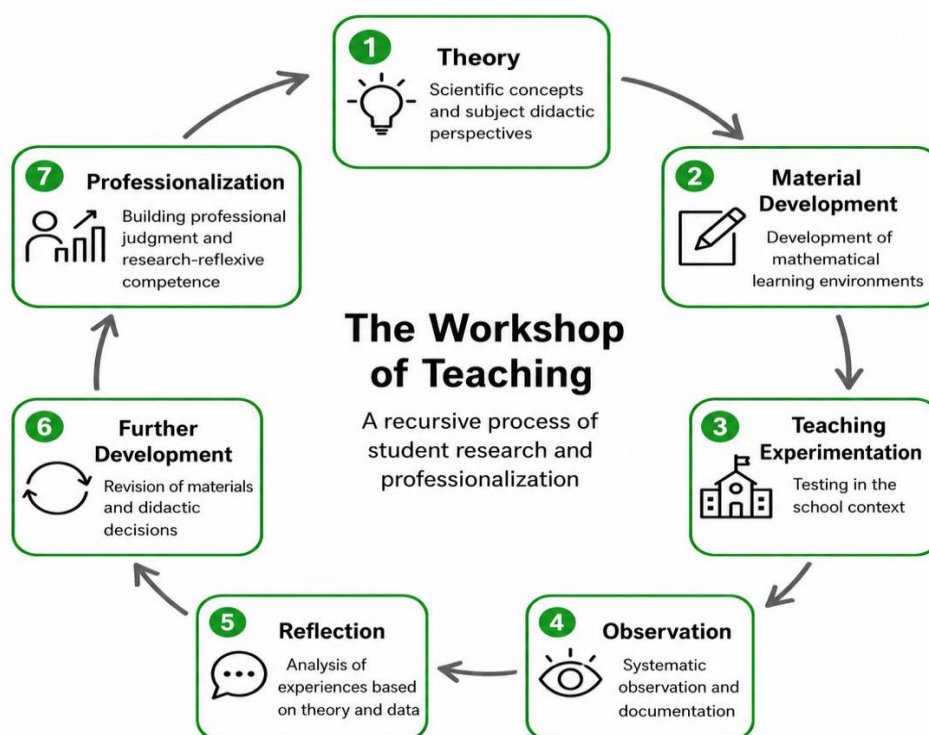


Figure 1: The Workshop of Teaching – a recursive process of student research and professionalization.

The circular representation illustrates that theoretical grounding, didactic development, practical trialling, observation and reflection do not form a linear sequence. Each reflection can initiate a new developmental cycle; Mathemundus represents one concrete realisation of this theoretical model.

The proposed conception is intended as a theoretical contribution to the further development of workshop-oriented teacher education. It does not claim to constitute a universally valid theory of professional development. Rather, it extends existing concepts through the idea of recursive professionalisation and invites further theoretical and empirical examination. Of particular interest is how the processes described here can be realised and developed in different subjects, workshop concepts and higher-education contexts.

Several implications follow for the design of teacher education. First, authentic school contact is most productive when it is prepared and followed by explicit theoretical work rather than positioned as an isolated practice component. Second, observation needs a clear focus: students require conceptual categories and shared questions if classroom experience is to become analytically productive. Third, revision should be built into the learning arrangement. If a designed learning environment is used only once, the central recursive movement from observation to renewed design remains incomplete. Finally, collaborative reflection requires time and facilitation so that different interpretations can be examined against subject-didactic concepts rather than merely exchanged as opinions.

The model also has limitations that should delimit its claims. It has been reconstructed from one university learning workshop in the field of primary mathematics education and is therefore context-bound. Its conceptual categories may be transferable, but their concrete realisation depends on curricular structures, access to schools, available time, staffing and the culture of collaboration between university and school. In addition, the present paper provides an initial conceptual reconstruction supported by accompanying student perceptions; it does not demonstrate causal effects on later teaching quality or pupils' learning. These limitations are not external to the model but point directly to the next research tasks.

Future research could examine recursive professionalisation longitudinally by following pre-service teachers across several workshop cycles and into school placements or early career teaching. Process-oriented analyses could investigate how didactic justifications change after classroom observation, which types of evidence students use when revising materials and how collaborative reflection influences professional judgement. Comparative studies across subjects and different university learning workshops would also help determine which elements of the model are context-specific and which constitute more general design principles of research-oriented teacher education.

9. Conclusion

The Workshop of Teaching conceptualises professionalisation as a continuous scholarly engagement with teaching. Academic knowledge, didactic decisions and practical experiences are related not additively but recursively.

Mathemundus illustrates how university learning workshops can support such a process: students develop learning environments, trial and observe them, reflect on their experiences through

theoretical perspectives and use the resulting insights for further development. Professional judgement thereby emerges in the interplay of academic theory, pedagogical practice and shared reflection.

The paper proposes the Workshop of Teaching as a conceptual starting point for the further development of workshop-oriented teacher education. The model invites university learning workshops to be understood as places of professional inquiry and calls for the recursive professionalisation processes described here to be investigated theoretically and empirically across different subjects and educational contexts.

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