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The “Smart Dreamcacher” (SD) Physical Particle Model II. Dream-World’s Mandala

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Abstract

In the previous part of this work (“Smart Dreamcatcher” (SD) Physical Particle Model...), we modeled the properties of commonly occurring hadrons described by charges and quantum numbers at the level of the 3-quark model of the 1960s and 1970s.

Following the chronology of the history of physics, we explore the properties of the basic units of the SD model that may make them suitable for visualizing the results of the "November Revolution" in particle physics.

With the simple rules of the "diametric replacement" procedure introduced here, we construct our own pseudo-universe, whose structural levels of arc-bit, arc-couple, and arc-chart can be mapped to the conceptual levels of quantum numbers, elementary particles (individual fermion particles), and particle families (quarks and leptons).

The entities of our model representing the system of fermions have no physical properties other than their pseudo quantum numbers. However, they carry these quantitative characteristics corresponding to the properties of physical elementary particles (hyper charge and isospin, electric charge, weak hyper charge and weak isospin, weak charge, chirality) in a necessarily true unity; and as a natural consequence, they require the presence of six quarks.

Within the framework of the model, the unequal occurrence of matter and antimatter can be interpreted, and to some extent it also reflects the distribution of matter and energy on the largest scale in the universe.

We point out that our empirical, conscious schemata of relational structures, which are ubiquitous in our everyday world, can be used to understand the structure of the Standard Model. Accordingly, we describe the system of quantum numbers characterizing fermions as a set of values of a three-variable function, as a partition of integers, as structures in Euclidean space, and as the topological properties of surfaces.

With the previous, current, and future parts of this work, our goal is to create an easily understandable construction that can be realized as a spatial, material, or virtual structure, which can be used to model elementary particles and their interactions.

We hope to provide a useful tool for teachers introducing young people to particle physics, which can serve as a starting point for studying the microcosm and ultimately help in the search for the ontological unity of the microcosm, macrocosm, and consciousness.

Introduction

In the first part of the paper **(1)**, we modeled the $\overline{quarks}$ of the 3-quark model of particle physics and the hadrons on an SD graph **(Figure 2)** constructed from 3-vertex hyper graph subunits (mini SD graph) **(Figure 1)**. We mapped the steps that can be taken between two vertices on the hyper edges of the SD graph to the quarks of the 3-quark model.

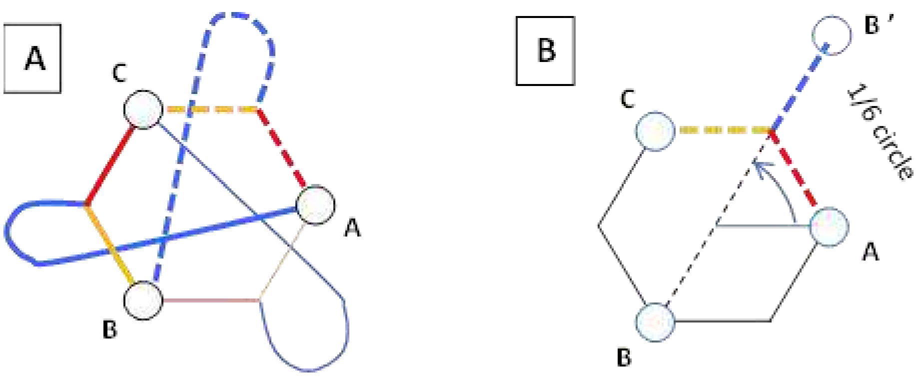


Figure 1. The mini SD-graph. **1/A**: The hyper edges (bold, thick, dashed) and vertices (A, B, C) of the mini SD-graph. The A-B step on the hyper edges (e.g. the step A -------- B on the dashed edge) takes y= -1/3 turn around the center.
1/B: An opened hyper edge (colored, dashed). One of the 3 ends of the dashed hyper edge moved from B to B’. The step A -------- B’ on the open hyper edge takes +1/6 turn around the middle (shown by an arrow).

The steps were characterized by a pair of values (y, i). (y) is the rotation around the axis of the SD graph (in circle units) associated with the step, and (i) is the radial distancing of the step, considering the period of the SD graph structure as a unit.

The (I) values correspond to the isospin charge of quarks. The (Y') values correspond to half of the hyper charge (Y) in the literature. The difference arises from the fact that we chose a full circle as the unit instead of π. We continue to use this unit, and an apostrophe indicates that the value shown is half of the hyper charge and weak hyper charge in the literature ($Y' = Y/2$ and $Y_W' = Y_W/2$).

The graph paths traversed along the hyper edges of the mini SD graph, whose starting and ending points are identical (cycles), symbolize hadrons.

We extended the stability condition on the SD graph to graph paths rotating $\pm k^* \pi$ around the axis of the SD graph (k = integer). These graph paths are SD model hadron particles if they cannot be further broken down into simpler graph cycles.

The ±k* 1/2 circle (k = 0, 1, 2) turn characteristic of SD baryons, partitioned into three members consisting of +1/6 and -1/3 components, represents the decomposition of stable graph paths (baryons) into quark steps, where the members of the partition determine the quantum number Y' of the quark steps. The other quantum number of the quark steps can be I= ±k*1/2, (k= 0, 1), following from the structure of the SD graph.

The SD quark steps are two-member partitions of the q= ±1/3 or q= ±2/3 in the form (Y'+I).

The opening of mini SD is the geometric equivalent of partitioning, which forms the complex system of SD quark steps from the identical quark steps of mini SD (Y' = ±1/3, I = 0), the quantitative properties of which become apparent during the traversal of the SD graph. The characterization of graph paths was given in the first part of the work **(1)**.

In this part, we analyze the structure of the mini SD graph and the consequences of its opening.

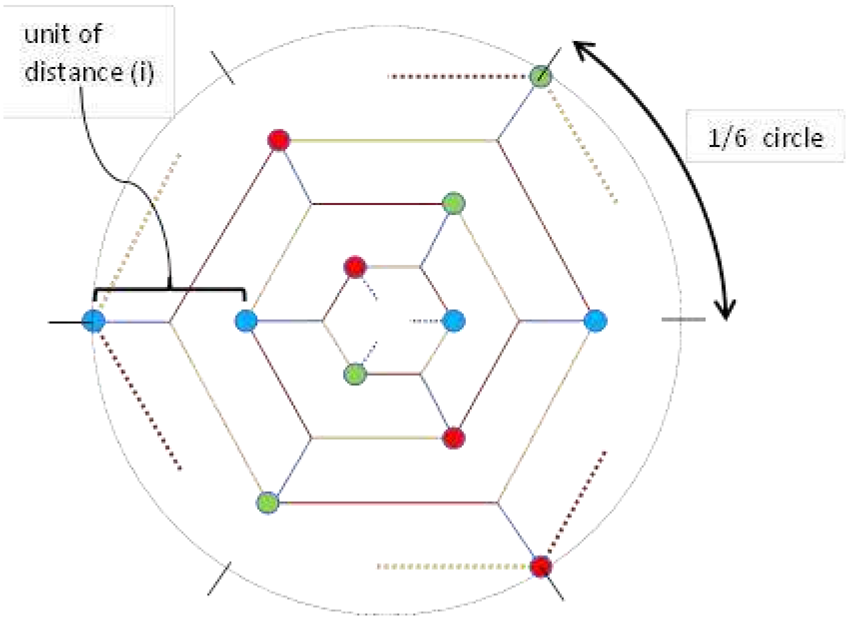


Figure 2. The SD graph (excerpt from **(1)**).

Results

SD prequark

The 3 hyper edges of the mini SD graph touch at vertices A, B, and C **(Figure 1/A)**. Traversing the 3 vertices of any hyper edge in the order A, B, C means three –1/3 rotations around the axis of the mini SD graph. The steps do not lead out of the plane of the mini SD graph. Thus, each step on the mini SD hyper edges is identical in terms of the properties (Y', I), and each step is characterized by the rotation Y' = –1/3 and the distancing I = 0. The steps of the mini SD graph are called SD **prequarks**, and the pair (Y' = –1/3, I = 0) is called the preduplet, which, after the graph is opened, become entities that can be characterized by properties corresponding to the quarks of the 3-quark model.

Choosing a system with opposite circulation, the steps are characterized by the values Y' = +2/3 and I = 0 (**alternative prequark**).

When opening one of the hyper edges of the mini SD graph, the endpoint touching edge B describes a lower or upper B – B' semicircle on a plane perpendicular to the mini SD plane **(Figure 1/B)**. The endpoint of the hyper edge will touch point B' outside the mini SD graph, which belongs to the next zone in the SD graph structure.

Thus, the A–B prequark step characterized by the values (Y' = –1/3, I= 0) becomes the A–B' step, and takes on the rotation and distance values (Y' = –1/3 + 1/2= +1/6, i= 0 ±1/2= ±1/2).

To analyze the opening of the hyper-edge, we resort to the conventions of representing reality in the art of ancient Egypt, we project the A–B step, the A–B' step, and the lower and upper semicircles B – B' of the perpendicular plane onto the same plane, as we see the fish in Nebamun's lake **(Figure 3)**. We represent the position and displacement of the endpoints of the hyper edge on the circumference of a circle.

Figure 3. Nebamun's pond (tomb-painting, Egypt 18. Dynasty ca. 1350 BC, British Museum, museum NR. EA37983) © The Trustees of the British Museum

Point of a circle is considered to be the endpoint of the arc drawn from a designated starting point 0 to the given point, and it is identified by the angle of rotation with the same sign.
Circular arcs drawn from the origin to a point in opposite directions are called **alternative arcs**.

Decomposition of the prequark – the birth of quantum numbers

To describe the correspondences between "SD prequark step → SD quark steps," we introduce a procedure called **diametric arc replacement**, which replaces an original arc a0 ($-2\pi \leq a0 \leq 2\pi$) with an ordered pair of arcs (a1, a2). We call a1 and a2 the diametric **arc bits** (a-bits) of a0. A diametric **a-duplet** is a tandem arc consisting of an ordered pair of a-bits (a1, a2), which leads from the starting point of the arc a0 to the end point of the arc a0.

<u>The rule of diametric arc replacement:</u>
(1) The arc a1 drawn from the point 0 of the circle to one endpoint of the diameter belonging to a0 and the consecutive arc a2 running into a0 are the diametric a-bits of a0 if they satisfy condition 2.
(2) None of the endpoints of a0, a1, or a2 is an internal point of a0 or (a1+a2).

One consequence of this rule is that the endpoints of the arcs a0, a1, and a2 lie on the same diameter of the circle.
As a first step, we perform the diametric replacement of the a0 = –1/3 arc characteristic of the prequark, as illustrated in **(Figure 4)**.

There are four diametric replacements of the a0 = –1/3 arc. The magnitude of the diametric a-bits of the a0 = +1/3 anti-quark is equal to these, but their sign is opposite **(Figure 5)**.
Note that the arc pairs (a1 = –5/6, a2 = ±1/2) and (a1= –1/3, a2 = ±1) are excluded from the diametric replacements of the arc a0 = –1/3. Although these tandem pairs of arcs lead from the starting point of the arc a0 = –1/3 to its endpoint, the endpoint of the arc a0 is an internal point of the composite arc (a1+a2), and therefore they violate the second condition of diametric replacement.

The endpoints of the +1/6 arc and the -5/6 arc are identical, but condition 2 excludes the a1= -5/6 arc, so the two directions are not equivalent in terms of diametric replacement.

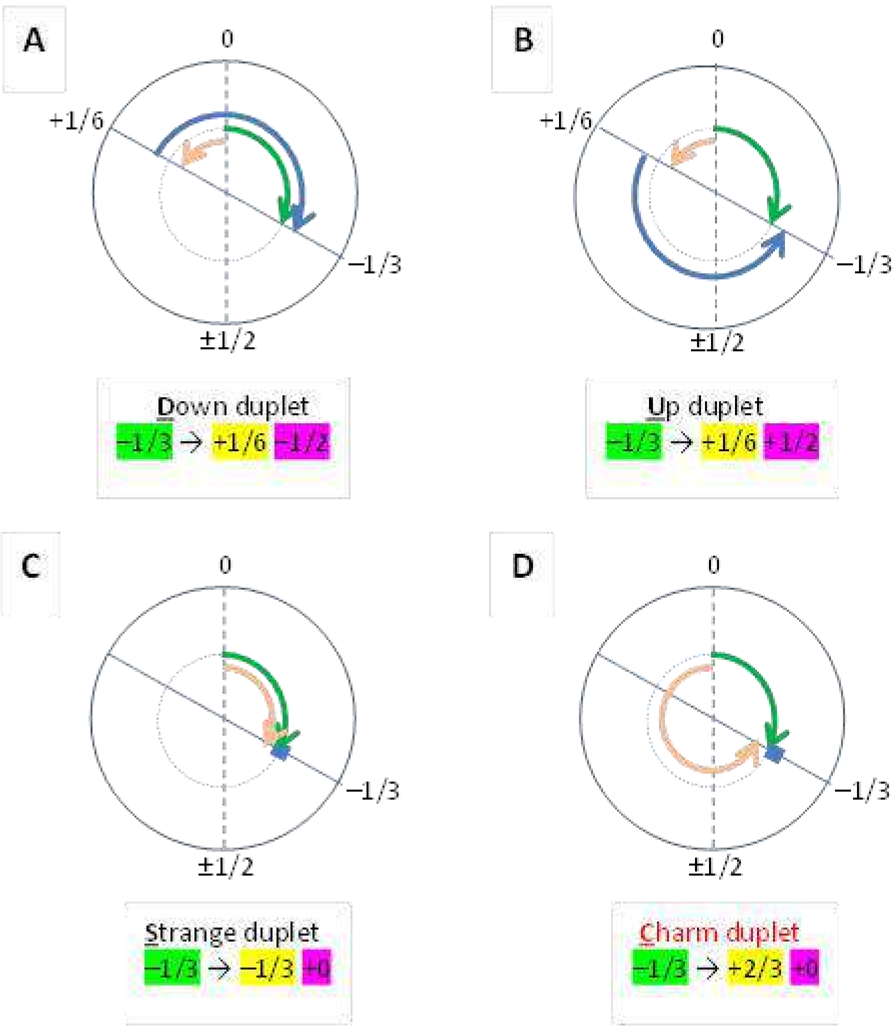


Figure 4. Diametric replacement of a0 = –1/3 (A, B, C, D).
The arc a0 = –1/3 (green) is replaced by arc a1 (orange) and a consecutive arc a2 (blue). The angle of a1 is marked on the circumference of the circle. Short radial arrows represent the a2-bits of 0 length. The rule for diametric replacement can be read in the text.

The quantum numbers of each quark (Y', I) in the 3-quark model correspond to one of the diametric replacements (a-duplets) of a0 = –1/3. The anti-a-bit pairs ($\overline{a1}$, $\overline{a2}$) of the +1/3 diametric replacement correspond to the quantum numbers of the anti-quarks **(Table 1)**.

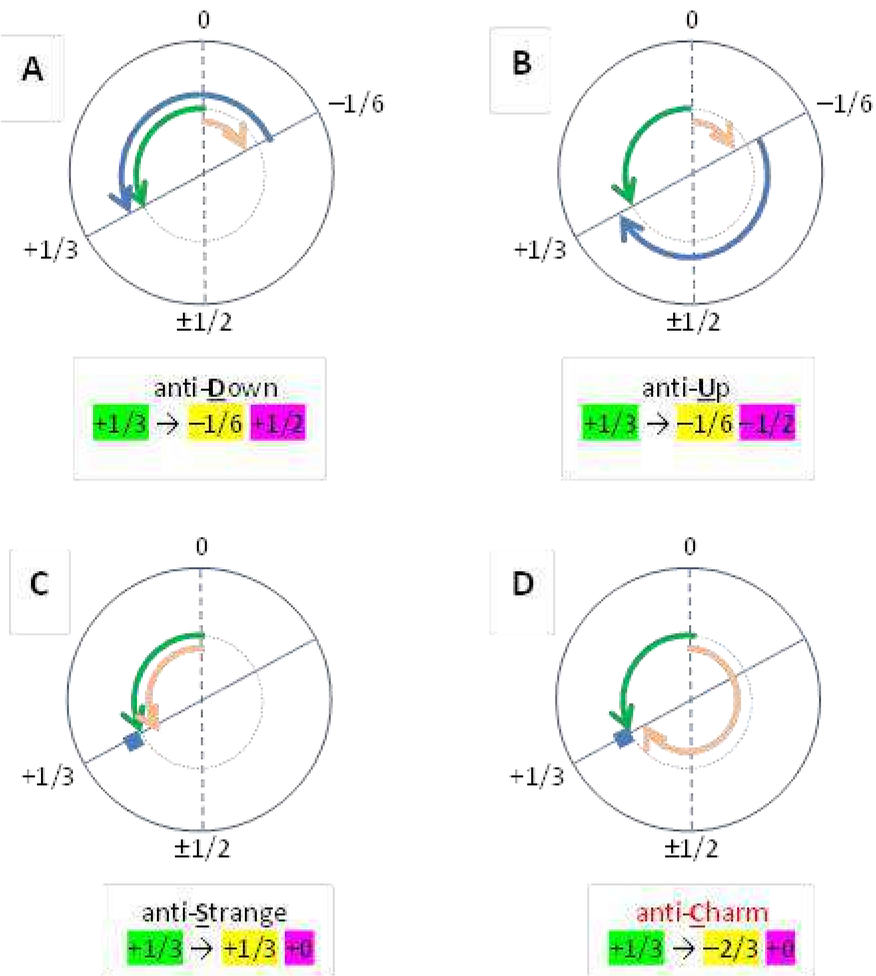


Figure 5. Diametric replacements of a0 = +1/3.
The arc a0 = +1/3 (green) is replaced by arc a1 (orange) and a consecutive arc a2 (blue). The angle of a1 is marked on the circumference of the circle. The angle of a1 is also marked if its endpoint is different from endpoint of a0. Short radial arrows represent the a2-bits of 0 length. The rule for diametric replacement can be read in the text.

Orphan quantum numbers

The a-duplet (a1= +2/3, a2= 0) and its anti-duplet (a1= –2/3, a2= 0) have no owner in the 3-quark model, which is the system of commonly occurring quarks. These orphan a-duplets correspond to the charm quark and charm anti-quark of the 4-quark model, whose quantum numbers are: (Y' = +2/3, I = 0) and (Y' = –2/3, I = 0).
The diametric replacements of the alternative prequark (a0 = +2/3) result in two new a-duplets (a1 = 2/3, a2= 0 and a1= -1/3, a2= 0), which correspond to the quantum numbers of the top quark and bottom quark of the 6-quark model. The topic of this paper is the 4-quark model, but we will return to the study of the top duplet and bottom duplet at another time.

Accordingly, we can assign the names of quarks and anti-quarks to the a-duplets (a-bit pairs), and we can also refer to the a-bits as **diametric quantum numbers**. In accordance with the physical analogy, the sum q = a1+a2 is the electric charge of the duplet.

Diametric (a1, a2)-bits of a0= −1/3			Hypercharge(Y') and isospin (I) of four quarks (Y' unit = 2π)		
a-duplet	a1.	a2	Quark.	Y'	I
down duplet	+1/6	−1/2	down quark	+1/6	−1/2
up duplet	+1/6	+1/2	up quark	+1/6	+1/2
strange duplet	−1/3	0	strange quark	−1/3	0
charm duplet	+2/3	0	charm quark	+2/3	0

Table 1. Diametric a-duplets. (a1, a2)-bits of a0 = –1/3 and a0 = +1/3 and the values of hyper charge (Y') and isospin (I) of the quarks and anti-quarks in the 4-quarks model.

The a-bits arranged in ascending order

The a-bits form part of an arithmetic sequence in which the difference is 1/6 **(Figure 6/A)**.

The a-bits can also be sorted into a 3×3 table, where the difference between the elements in the columns is 1/6 and the difference between the elements in the rows is 1/2 **(Figure 6/B-C-D)**.

The a-duplets and anti-duplets have the same a2 value set, but their a1 value sets are different. The first row of the table contains the a1 values, the third row contains the $\overline{a1}$ values, and the second row contains the a2 and the corresponding $\overline{a2}$ values.

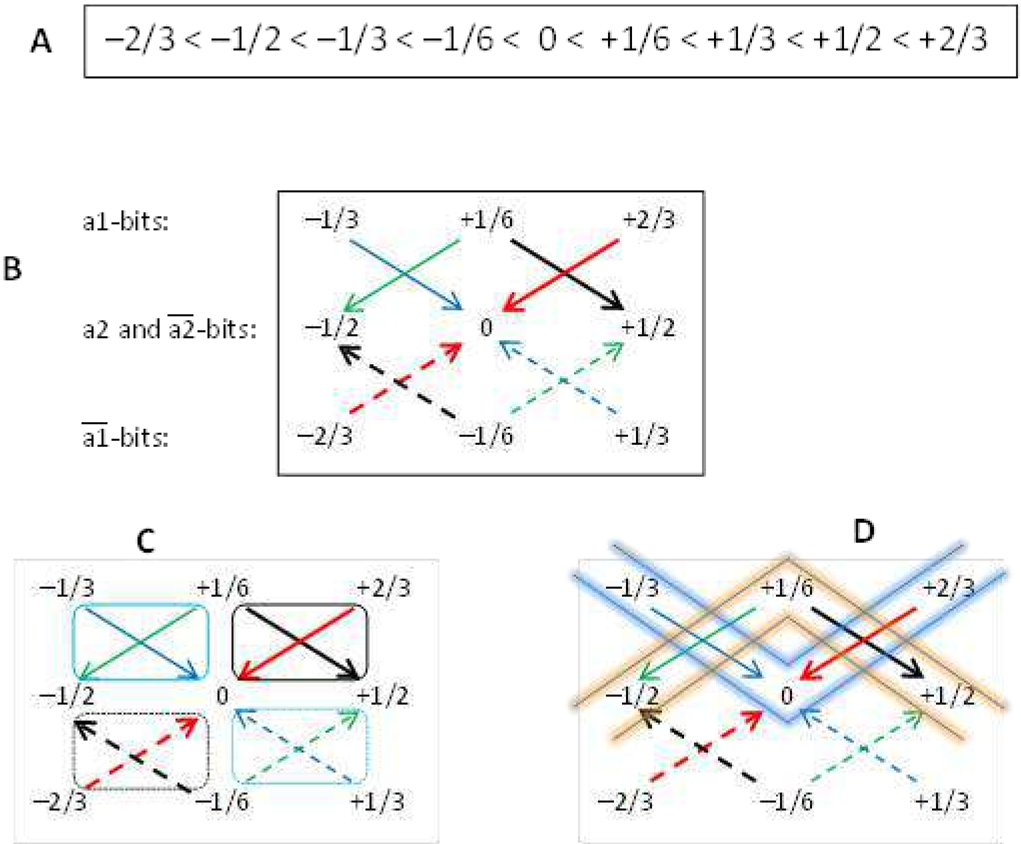


Figure 6. Sorting the diametric a-bits.
The series of the a-bits **(Figure 6/A)**.
The quark chart containing the diametric a-bits of a0 = -1/3 and a0 = +1/3 **(Figure 6/B-C-D)**.
The a1-bit and a2-bit of a-duplet are connected by colored arrows. Green: down duplet, blue: strange duplet, black: up duplet, red: charm duplet. Their anti-duplets are connected by dashed arrows of same color.
Figure 6/B indicates the rows of a1-bits, $\overline{a1}$-bits and a2/$\overline{a2}$-bits. The quark duplets of equal q = a1+a2 are framed **(Figure 6/C)**. The generations of quark duplets are framed **(Figure 6/D)**.

We have connected the a-bits of the a-duplets and the $\bar{a}$-duplets **(Figure 6/B)**. We will refer to this figure as a **quark chart**.

The upper half of the quark chart contains the quark duplets, and the lower half contains the $\overline{\text{quark}}$ duplets. The chart is named after duplets in the upper half of it, so the [–]-reflected quark chart is called the $\overline{\text{quark}}$ chart.

Symmetries of the chart

In the following, we use abbreviations to denote the geometric reflections of n × n tables.

Reflection across the horizontal central axis is denoted by [–], reflection across the vertical central axis by [ı], reflection across the main diagonal by [\], reflection across the secondary diagonal by [/], and reflection through the center by [·]. The symbol for a transformation leaving the table unchanged is [#].

The central line of the symbols [–], [ı], [·], [/], [\] evokes the direction of the reflection axis.

An asterisk (*) is placed between the symbols of successive transformations.

Reflection across the horizontal and vertical axes as well as reflection across the main diagonal and the secondary diagonal yields the same result as reflection across the center: [–]* [ı] = [\]*[/] = [·]. Reflection are commutative transformations.

Similar to quarks, we divide a-duplets and $\bar{a}$-duplets into generations. Duplets of the same generation are [–]-symmetrical on the chart **(Figure 6/D)**.

Duplets with the same q = (a1+a2) are shown as intersecting arrows on the chart **(Figure 6/C)**.

As a consequence of the a-bits' serial properties, the [–]-reflection gives the value -1/3 to the a1 values (row 1 of the chart) and gives the value +1/3 to the $\overline{a1}$ values (row 3 of the chart). The [ı]-reflection gives the value +1 to the values in column 1 and gives the value -1 to the values in column 3 of the chart.

The duplets and their corresponding anti-duplets are [·]-mirror images of each other on the quark chart. Thus, duplets and anti-duplets belonging to the same generation can be transformed into each other by reflection. For example, the [ı]-reflection of a $\bar{d}$-duplet results in a $\bar{u}$-duplet, whose [–]-reflection results in a d-duplet. The [ı]-reflection causes a change of Δq= ±1, while the [–]reflection causes a change of Δq = ±1/3. The [·]-reflection changes the sign of the a-bits to the opposite.

Due to the properties of reflection, a reflection performed twice consecutively, or all three reflections performed, will return the original duplet.

C.I.A.O. relations

Between points A and B on a line there are two directed paths (A_B and B_A), which can have two different relations: (A_B) is equal to (A_B), (B_A) is equal to (B_A), and (A_B) is opposite to (B_A), (B_A) is opposite to (A_B).

The arcs connecting two points of a circle can have the same or opposite direction, and either of the two points can be the starting point or the end point of a connecting arc.

Accordingly, there are four connecting arcs, and they can relate to each other in four ways:

1. Arcs with the same direction and the same starting point are **identical** (**ide**),
2. Arcs with the same direction and different starting points are **complementary** (**com**),
3. Arcs with opposite directions and the same starting point are **alternative** (**alt**),
4. Arcs with opposite directions and different starting points are **opposite** (**opp**).

So, two points on a circle are connected by four distinct arcs: two alternative arcs and their opposites. We called these four symmetrical relations C.I.A.O. relations, and arcs in the relation with an arc (x) are denoted as (x)ide, (x)com, (x)alt, (x)opp, where $-1 \le x \le +1$. Their connections are illustrated in **(Figure 7)**.

An arc of length x = 0 has no direction, so the relations (0)ide, (0)com, (0)alt, (0)opp are valid by distinguishing -0 and +0: (-0)ide = -0, (-0)com = -1, (-0)alt = +1, (-0)opp = +0, and (+0)ide = +0, (+0)com = +1, (+0)alt = -1, (+0)opp = -0.

For the direction and magnitude of the arcs in the relation, it holds that (x)ide = - (x)opp and (x)com = - (x)alt.

There is a point in both directions at an arbitrary distance x from point 0 on the line. These points correspond to points on a circle at an angular distance ±1/x from point 0. Accordingly, the C.I.A.O. relations are assigned the algebraic expressions

$R_{ide} = +(1/x \pm 0)$, $R_{opp} = -R_{ide} = -(1/x \pm 0)$, $R_{alt} = +(1/x \pm 1)$, $R_{com} = -R_{alt} = -(1/x \ \pm 1)$

in which the second term of the expression must be chosen to have the opposite sign to x. These are called the R-expressions of the relation and give the coordinate value of the endpoint of the arc on the circle that is in the indicated relation with the arc 1/x.

C.I.A.O. relations can also be interpreted as single-variable operations that produce a pair given by the R-expression for every 1/x point on the circle.

Performing repeatedly, the operation x → ide(x) returns x. Alongside the x → (x)ide operation, x is returned if either of the operations x → (x)com, x → (x)alt or x → (x)opp is applied twice, or if all three operations are performed.

The arcs connecting two points on the circle form a relational structure similar to the [–], [ı], [·] and [/], [\], [·] transformations of a table. By performing any of them twice, or by performing all three transformations, we get back the original table.

The results of consecutive operations are illustrated by the traversal of the Com, Ide, Alt, Opp (CIAO) 2D and 3D grids **(Figure 7/C, D)**.

The equality (x) * alt * opp * com = (x) * ide still holds if we arbitrarily rearrange the R operations between the two sides. So, no matter how we split the set of relations into two disjunct parts, the operations of the two subsets give the same result to any x ($-1 \le x \le +1$).

The sign of multiplication here means performing the appropriate R operation on x, then applying the next R operation on the result of the operation, and so on.

A	Identical	Opposite	Alternative	Complementer
Identical	ide	opp	alt	com
Opposite	opp	ide	com	alt
Alternative	alt	com	ide	opp
Complementer	com	alt	opp	ide

B X = +1/3	Identical	Opposite	Alternative	Complementer
(x)ide	+1/3	−1/3	−2/3	+2/3
(x)opp	−1/3	+1/3	+2/3	−2/3
(x)alt	−2/3	+2/3	+1/3	−1/3
(x)com	+2/3	−2/3	−1/3	+1/3

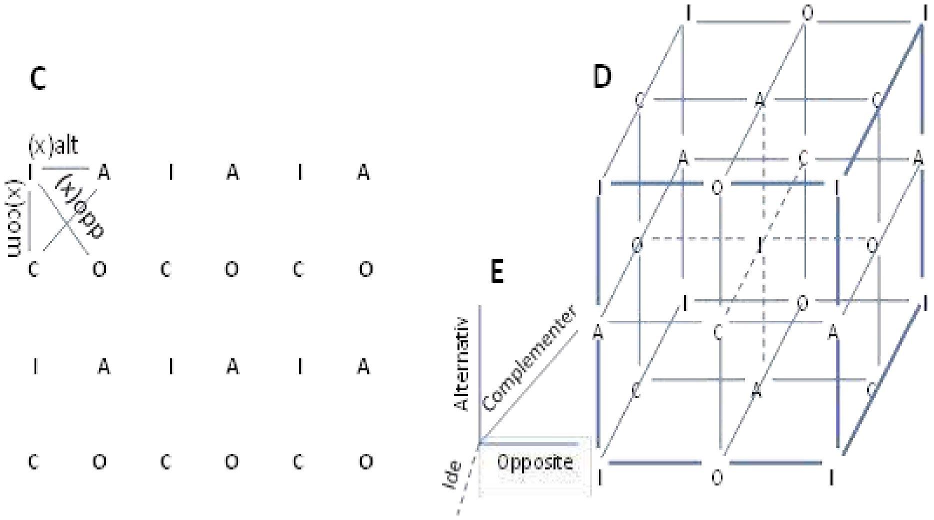


Figure 7. Tables of operations (x)ide, (x)opp, (x)alt, (x)com. Results of two consecutive operations (**A**). Results of two consecutive operations in case x = +1/3 (**B**).
Part of a CIAO lattice of operations com, alt, opp (**C**). Initial of the operation represents the result of the operation performed on x. Horizontal, vertical or diagonal bar between any of two neighboring initials represents an operation assigned to the bar. The bar joints the input to the output of the operation. A part of the 3D CIAO grid (**D**) and its coordinates (**E**). The 3D diameters corresponding the ide relation are not shown.

The diametric replacement of the a0 = 0 arc (replacement of "nothing")

The endpoint of the prequark arc a0 = -1/3 is created by a rotation of -1/3. The original state can be restored by rotating the point a0 back to the point a0 = 0.

On the circle, we can also achieve this with the operations (-1/3)opp = +1/3 and (-1/3)com = -2/3.

The anti-prequark a0 = +1/3 is brought to the state a0 = 0 in the same way, (1/3)opp = -1/3 and (1/3)com = +2/3.

The applied diametric rotations (+1/3, -1/3, +2/3, -2/3) are in the ide, com, alt, opp relations with each other **(Figure 8)**.

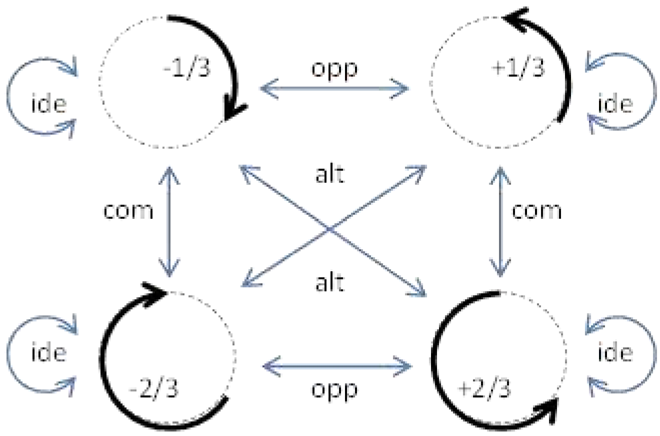


Figure 8. Relations between arcs -1/3, +1/3, -2/3, +1/3.

The a0 = 0 arc created in this way can also be decomposed into (a1, a2)-bits according to the rules of diametric replacement. Considering that the endpoints of the a0, a1, a2 are located on the same diameter, the a-bits of the a0 = 0 arc are obtained from the a-bits of the a0 = –1/3 arc by adding the value +1/3 to the a1-bits. This can be achieved by rotating the diameter –1/3__+1/6 containing the endpoints of the arcs a1 and a2 to the diameter 0__+1/2.
The diametric ($\overline{a1}$, $\overline{a2}$) bits of the a0 = 0 are obtained from the diametric ($\overline{a1}$, $\overline{a2}$) bits of the a0 = +1/3 by adding the value –1/3 to the $\overline{a1}$ values. This means rotating the +1/3__–1/6 diameter to the 0__+1/2 diameter.
According to the rules of replacement, the a2 bits lead from the endpoint of the circle diameter to an endpoint of the same diameter. Obviously, rotating the diameter does not change their size. Therefore, we leave the value of the a2 bits unchanged.

Thus, the diameter +1/6__–1/3 containing the endpoint a0 = –1/3 is transformed into the diameter ±1/2__0 by the alternative rotations +1/3 or –2/3, and the diameter –1/6__+1/3 containing the endpoint a0 = +1/3 is transformed into the diameter ±1/2__0 by the alternative rotations –1/3 or +2/3. **(Figure 9)**.

We named a diametric [−]-rotation of the chart ([−]-D_{turn} = x) if we add opposite x values to the a-bits of the first and last rows of the chart, while leaving the middle row of the chart (a2 and $\overline{a2}$ a-bits) unchanged. We named the diametric [|]-rotation of a chart ([|]- D_{turn} = x) if the a-bits of the first and last columns are given opposite x values, while the middle column of the chart is left unchanged.
The diametric replacement of a0 = 0 therefore corresponds to diametric rotations ([−]-D_{turn} = +1/3 and [−]-D_{turn} = -2/3) of the quark chart.

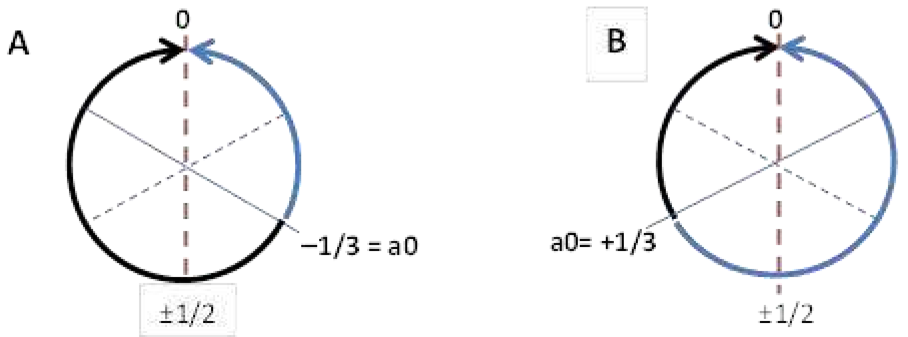
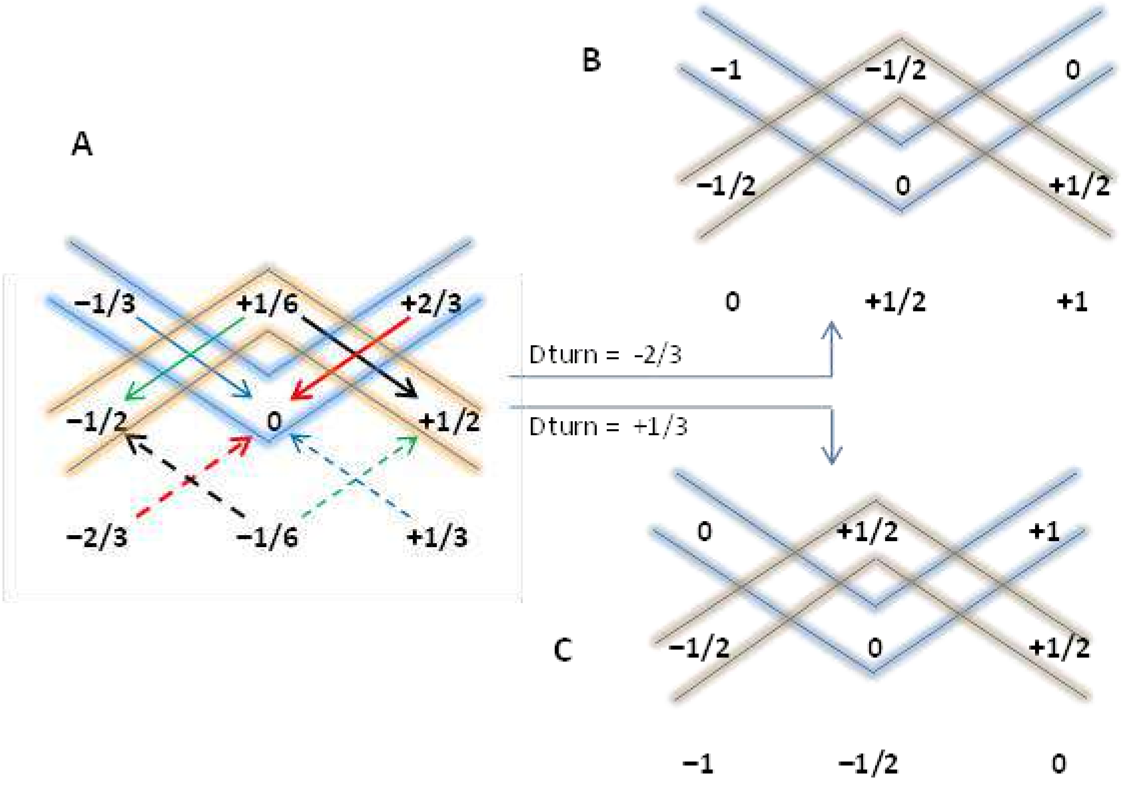


Figure 9. Rotation of the diameters. **A**: Rotation of the diameter +1/6__–1/3 to the diameter ±1/2__0 by +1/3 (blue arc) and by -2/3 (black arc). **B**: Rotation of the diameter –1/6__+1/3 to the diameter ±1/2__0 by –1/3 (black arc) and by +2/3 (blue arc). The diameter is a continuous straight line before rotation, while the rotated diameter is a dashed red line.

By diametric rotation D_{turn} = -2/3 of the quark chart **(Figure 10/A)**, we obtained a new chart **(Figure 10/B)**. The quark duplets are transformed into duplets in which the value of the a-bits is equal to the weak hyper charge and weak isospin quantum numbers of the lepton and anti-lepton particles.
The first row of the lepton chart contains the diametric a1 bits of the a0 = 0 arc, which correspond to the weak hyper charge of leptons. The third row contains the diametric $\overline{a1}$ bits of the a0 = 0 arc, which correspond to the weak hyper charge of anti-leptons. The middle row of the chart contains diametric a2 bits of the a0 = 0, which are the weak isospin charges of leptons and leptons.

The new chart is called the $\overline{\boldsymbol{lepton}}$ **chart (Figure 10/B)**. In the SD model, hyper charge corresponds to a turn, and as in the previous part of the work, we have chosen the full circle as the unit of spin instead of π. Therefore, the value of a1 (weak hyper charge) is half of the commonly used values, a1 = $Y_{weak}/2$.
Adding the value of ±1/3 to the a1/a1 bits of the quark chart produces another chart (**lepton chart**), which is a [−]-mirror image of the lepton chart, i.e., the lepton a1 bits and the lepton $\overline{a1}$ bits have been swapped **(Figure 10/C)**. **Figure 10/A**, **Figure 10/B**, and **Figure 10/C** are diametric rotations (D_{turn} = 0, D_{turn} = +1/3, D_{turn} = -2/3) of the quark chart.



D

	+2/3	+1/6	-1/3	-5/6
-5/6	-3/2	-1	-1/2	0
-1/3	-1	-1/2	0	+1/2
+1/6	-1/2	0	+1/2	+1
+2/3	0	+1/2	+1	+3/2

E

	+2/3	+1/6	-1/3	-5/6
+2/3	0	+1/2	+1	+3/2
+1/6	-1/2	0	+1/2	+1
-1/3	-1	-1/2	0	+1/2
-5/6	-3/2	-1	-1/2	0

Figure 10. Lepton chart (**B**) and lepton chart (**C**) generated from quark chart (**A**) by diametric rotations +1/3 and -2/3. Lepton chart (**D**) and lepton chart (**E**) generated as difference matrices (framed) of quark chart’s a1-bits (row and column heads of the matrices).

The coordinates of the endpoints of the circle’s diameter 1/3__+1/6 and the alternative coordinates (+2/3, +1/6, -1/3, -5/6) are the a1-bits of the quark chart, of which the value -5/6 is excluded by the rule of diametric replacement. Their difference matrix **(Figure 10/D, E)** is identical to the lepton chart and the lepton chart.

From each quark duplet, the diametric rotation D_{turn} = 1/3 and D_{turn} = -2/3 produces two [−]-symmetric lepton/lepton duplets of the lepton chart. Furthermore, the same lepton/lepton duplet is formed from two [−]-symmetric quark/quark duplets. The quark/quark duplets and the lepton/lepton duplets transformed from them are listed in the same rows of **Table 2**.

The electric charge (q) of lepton/lepton duplets, similarly to the charge q of quark/quark duplets, is interpreted with the expressions q = a1+a2 and q = a1+a2.

Sources			a-duplets generated by diametric rotation					
Quark duplets a0 = −1/3			Lepton duplets generated by −2/3			Anti-lepton duplets generated by +1/3		
	a1	a2		a1	a2		a1	a2
d	+1/6	−1/2	Le	−1/2	−1/2	R$\overline{v}$	+1/2	−1/2
u	+1/6	+1/2	Lv	−1/2	+1/2	R$\overline{e}$	+1/2	+1/2
s	−1/3	0	Re	−1	0	L$\overline{v}$	0	0
c	+2/3	0	Rv	0	0	L$\overline{e}$	+1	0
Anti-quark duplets a0 = +1/3			Lepton duplets generated by −1/3			Anti-lepton duplets generated by +2/3		
	a1	a2		a1	a2		a1	a2
$\overline{d}$	−1/6	+1/2	Lv	−1/2	+1/2	R$\overline{e}$	+1/2	+1/2
$\overline{u}$	−1/6	−1/2	Le	−1/2	−1/2	R$\overline{v}$	+1/2	−1/2
$\overline{s}$	+1/3	0	Rv	0	0	L$\overline{e}$	+1	0
$\overline{c}$	−2/3	0	Re	−1	0	L$\overline{v}$	0	0

Table 2. The a-bits of quark/$\overline{\text{quark}}$ duplets and the corresponding lepton/$\overline{\text{lepton}}$ duplets created by diametric rotations –1/3, +2/3, +1/3, –2/3.

We must not forget that the properties of charts and the relationships between charts all stem from the simple fact that the a-bits of quark duplets form an arithmetic series where the difference is 1/6. This leads to seemingly complicated relationships, but they can always be traced back to this basis.
The self-imposed development of our model apparently confused two qualitatively different levels of physical reality. For the 4-quark, it produced the quantum numbers Y’ and I operating in the strong interactions. However, it produced the quantum numbers Y'_W and I_W, which are necessary to describe the weak interactions of leptons. This incompatibility will be clarified at the end of the paper.

Considering the relational structure of the diametric rotations -1/3, +1/3, -2/3, +1/3 that unify the quark duplets and lepton duplets **(Figure 8)**, let us think back 500 years to Bruder Klaus **(1)**, who was fascinated by studying the unified whole manifested in his wheel divided into three parts. For him, the three spokes converging in the center of the wheel symbolized the superior forces governing the world, which “have embraced heaven and the whole world” and “they are one and undivided in eternal rule”.
The wise hermit, pondering the secrets of the three chipped spokes of the wheel, thought he saw the embodiment of the world's deep connections, which he tried to put into words in his own way — just like we do.

Charts arranged in ascending order

From the arrangement of the a-bits, and the method of generating the lepton chart, it follows that the lepton chart, quark chart, quark chart, lepton chart form a row with a 1/3 difference **(Figure 11/A)**. The a-bits of the upper row of consecutive charts increase from left to right, while those of the lower rows increase from right to left by 1/3.
The ordered row of four charts can be represented on the circumference of a circle, forming a **chart circle (Figure 11/B)**.

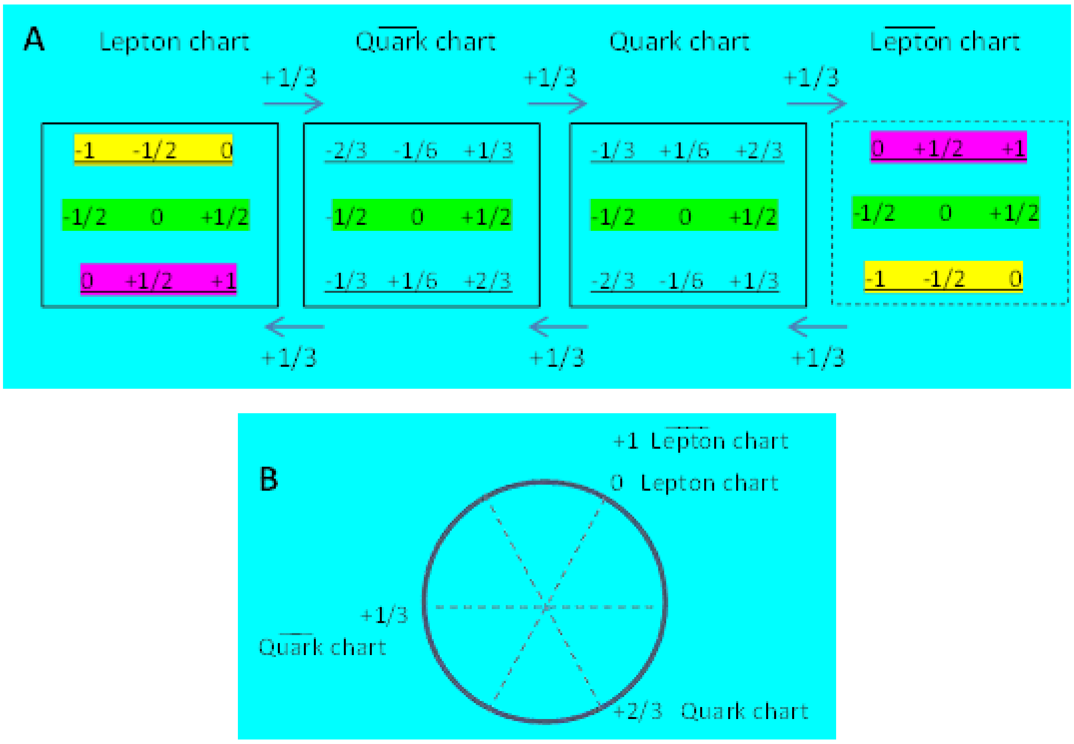
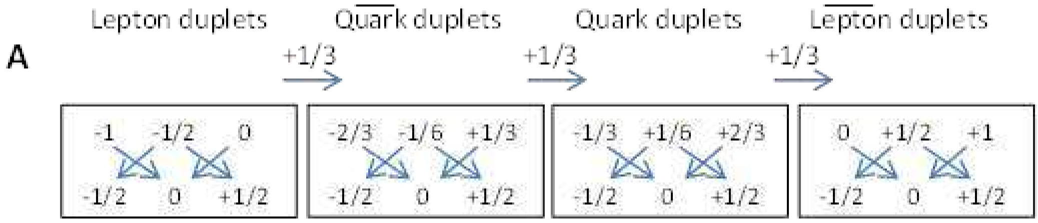


Figure 11. Arrangement of the lepton chart, $\overline{\text{quark}}$ chart, quark chart, and $\overline{\text{lepton}}$ chart in a row (A) and around a circle (B).

The top and bottom halves of the arranged charts form two identical but oppositely increasing sequences **(Table 3/A)** and reflect the differences between the a1-bits of lepton-, $\overline{\text{kvark}}$-, quark-, and $\overline{\text{lepton}}$ duplets. As a result, the data from **(Table 2)** can be condensed into four columns **(Table 3/B)**.

The four columns of **Table 3/B** contain the a-bits of the lepton, anti-quark, quark, and anti-lepton duplets. The rows of the table are composed of duplets generated from each other by diametric rotation.

(Table 3/B) also illustrates that a lepton/$\overline{\text{lepton}}$ duplet pair generated from two quark/$\overline{\text{quark}}$ duplets of the same generation but different up/down types ([–]-symmetric duplets of the charts) are identical.



B

$D_{turn} = 0$			$D_{turn} = +1/3$			$D_{turn} = +2/3$			$D_{turn} = +1$		
Lepton duplets			$\overline{\text{Quark}}$ duplets			Quark duplets			$\overline{\text{Lepton}}$ duplets		
	a1	a2		a1	a2		a1	a2		a1	a2
Le	−1/2	−1/2	$\overline{u}$	−1/6	−1/2	d	+1/6	−1/2	R$\overline{v}$	+1/2	−1/2
Lv	−1/2	+1/2	$\overline{d}$	−1/6	+1/2	u	+1/6	+1/2	R$\overline{e}$	+1/2	+1/2
Re	−1	0	$\overline{c}$	−2/3	0	s	−1/3	0	L$\overline{v}$	0	0
Rv	0	0	$\overline{s}$	+1/3	0	c	+2/3	0	L$\overline{e}$	+1	0

Table 3. The top halves of the charts arranged in a row (A) and the duplets contained within them (B). In the charts (A), duplets are indicated by a1→a2, and their list is shown below the corresponding chart (B).

The a-bits and a-duplets in the α, β, γ coordinate system

The properties of the sequence of a-bits arranged on the charts **(Figure 6)** imply that the lepton chart can be considered as a two-dimensional α, β coordinate system.

The a-duplets are made up of two a-bits, and each a-bit has two coordinates (α and β), where α = (–1, 0, +1), β = (–1, 0, +1) **(Table 4)**. The β coordinate of the a1-bits is β = 1, and the coordinate of the a2-bits is β= 0. The β coordinate of the $\overline{a1}$-bits is –1.

The value of the a-bit at point α, β of the coordinate system is (α + β)/2.

The values of a-bits of lepton charts and quark charts can be described by a common term if we introduce the variable γ. For lepton charts γ = 1, for quark charts γ = 1/3.

The a-bits are the values of the three-variable function Ⅎ(α,β,γ) = (α + β*γ)/2, where –1 ≤ α ≤ 1 and –1 ≤ β ≤ 1, integer.

We will refer to the expressions generating the values of the a-bits on the charts with the α, β coordinates as **chart-expressions**.

The lepton chart is located at the value γ = 1 in the α, β, γ coordinate system of **Figure 12**, where we also depicted charts corresponding to other values of γ.

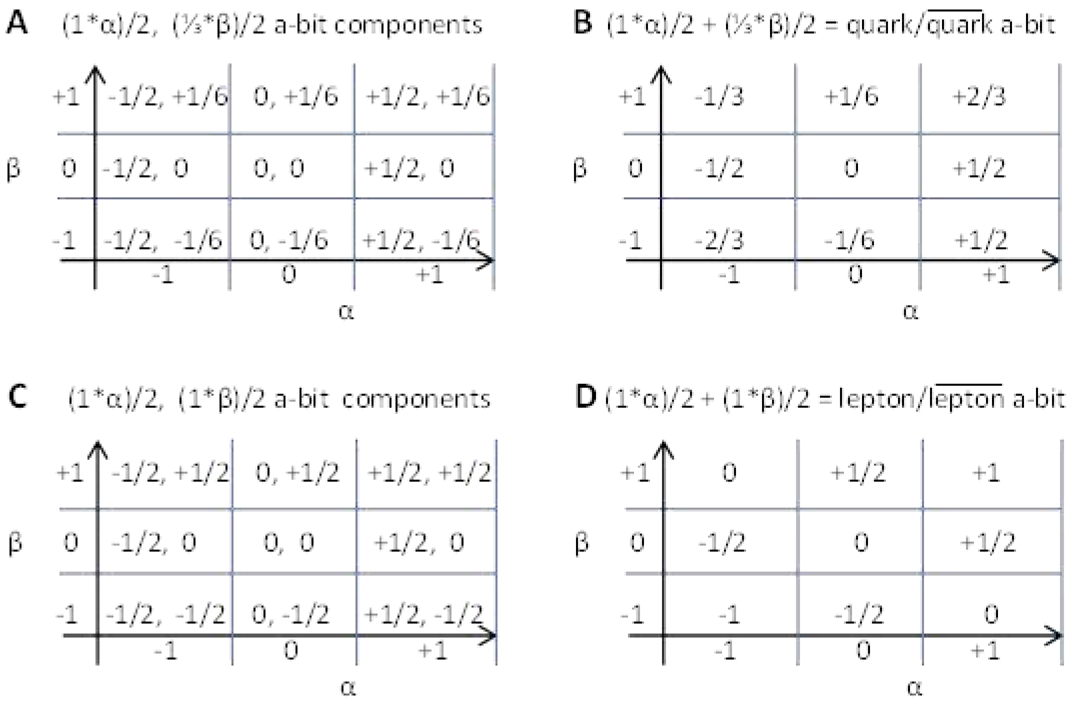


Table 4. Components (1*α)/2 and (⅓*β)/2 of the quark a-bits (A), sum of the components (B), components (1*α)/2, (1*β)/2 of the lepton a-bits (C), sum of the components (D) in the α, β coordinate system.

Every value pair in the range of the Ⅎ(α,β,γ) function corresponds to two a-bits (two quantum numbers) of a fermion duplet if the difference between the two coordinates α and between the two coordinates β is also ±1. Those and only those a-bits form an a-duplet whose α coordinates and β coordinates differ by ±1. This property is the common definition of quark and lepton duplets in the α, β coordinate system and can be read off from the charts.

The value of the a-bits of charts is given by the expression (α + γ*β)/2, and the value of the a-bits of anti-charts is given by the expression (α – γ*β)/2. For the lepton chart and $\overline{\text{lepton}}$ chart, γ = 1, so their a-bits can be divided into an (α/2) and a (β/2) component, and their values are calculated by averaging the (α) and (β) coordinates. (α+β)/2 is the arithmetic mean of the (α) and (β) coordinate values, while (α–β)/2 is the deviation of α and β from their arithmetic mean. Using the Pythagorean theorem to the geometric mean, arithmetic mean, and deviation from the mean of two values: $((\alpha+\beta)/2)^{1/2} = ((\alpha-\beta)/2)^{1/2} \pm \alpha*\beta$, where α*β is the square of the geometric mean of the two values. The appropriate option must be chosen from the ± options.

For the quark chart and $\overline{\text{quark}}$ chart, γ = 1/3. Therefore, their a-bits can be divided into an (α/2) and a (β/6) component. The relationship of their a-bits can be written in the generalized form $((\alpha + \gamma*\beta)/2)^{1/2} = ((\alpha - \gamma*\beta)/2)^{1/2} \pm \alpha*\beta*\gamma$, analogous to the Pythagorean theorem.

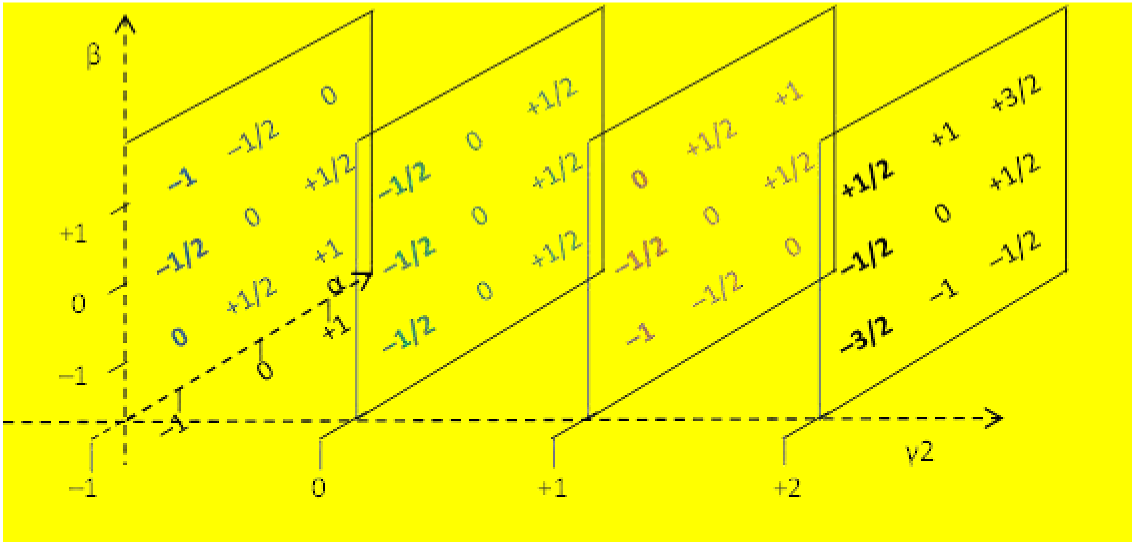


Figure 12. Lepton chart variants in an α, β, γ coordinate system. The value of an a-bit is (α + β*γ)/2, (-1 ≤ α ≤ 1, -1 ≤ β ≤ 1, integer). γ = 1 for the lepton chart. The quark chart is located at the value γ = 1/3 (data not shown).

From the table **(Table 4)** it can also be seen that the electric charge q = a1 + a2 of every quark duplet can be decomposed into a 1/6 component and a ±1/2 component. The 1/6 component is always contained in the a1-bit, while the ±1/2 component appears in one of the two a-bits. Similarly, the two components of lepton duplets are 1/2 and ±1/2. In anti-duplets, the signs are opposite.

Thus, every quark/quark duplet consists of one ±1/6 and one ±1/2 component, while the two components of lepton/lepton duplets are ±1/2 and ±1/2. In other words, the two a-bits forming the duplets are connected to each other by the ±1/6 component, or by the 3* ±1/6 = ±1/2 component.
Eliminating the 1/2 factor of the chart expression, we use the forms Γ1 = ½*γ1 and Γ2 = ½*γ2 in the expressions, which are thus of the form Γ1*α + Γ2* β.

α and β can only have the values -1, 0, +1, therefore the a-bits of the charts can also be expressed with the values Γ1 and Γ2 **(Table 5)**. As a result, we built the a-bits and duplets from ±1/2 and ±1/6 components. Where α and β previously took care of the proper setting of the signs, now we provide them directly to Γ1 and Γ1.
If we allow the value Γ1 = 0 in addition to the value Γ1 = 1/2 characteristic of fermion duplets, then the 0* α+ Γ2* β chart expressions define two new charts **(Table 5/C)**.
If we allow the value Γ2 = 0 in addition to the values Γ2 = 1/6 characteristic of quark duplets and Γ2 = 1/2 characteristic of lepton duplets, then the Γ1*α + β* 0 chart expressions define two new charts **(Table 5/D)**.

A

Chart expression: Γ1*α + Γ2 *β Generalized chart		
-Γ1 + Γ2	0 + Γ2	Γ1+ Γ2
-Γ1 + 0	0	Γ1+ 0
-Γ1 -Γ2	0 - Γ2	Γ1 - Γ2

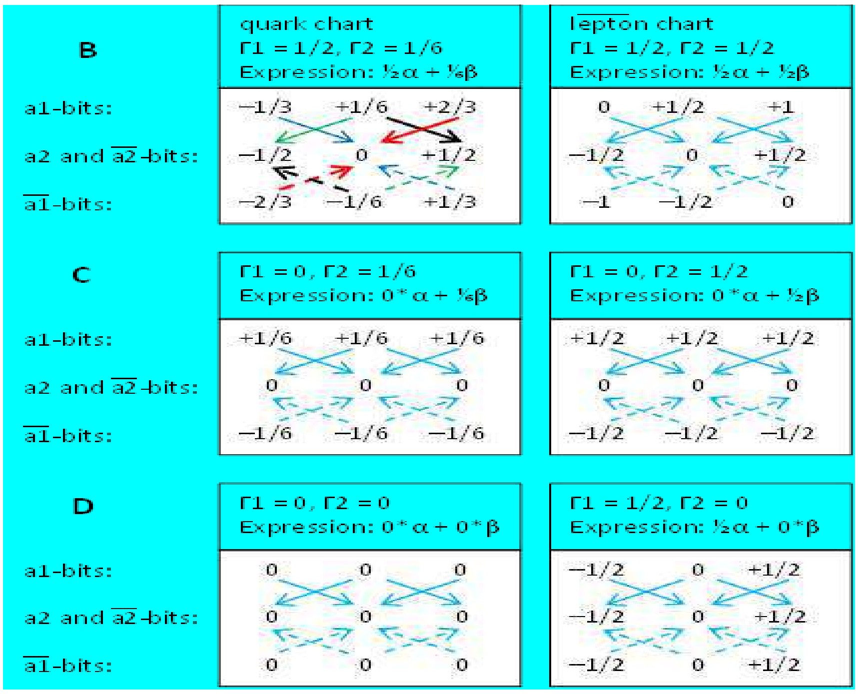


Table 5. The a-bits of the fermion/fermion chart expressed with Γ1 and Γ2 coefficients **(A)**. The quark chart and the lepton chart **(B)**, as well as four charts generated with coefficients Γ1 = 0 and Γ1 = ½ **(C, D)**. The headers of the charts indicate the values of Γ1 and Γ2 and the chart expression.

With the α and β coordinates, the [−]-Dturn = x and [ı]-Dturn = x of the chart can be formulated by adding x* β and x* α values to the a-bits, respectively.

The 0*α + ⅙β chart is set by [−]-Dturn = +1/6, the 0*α + ½β chart is set by [−]-Dturn = +1/2, and the ½α + 0*β chart is set by [ı]-Dturn = +1/2 from the 0*α + 0*β chart.

Applying the rules of matrix addition to the charts, the 0*α + ½β chart is the sum of three 0*α + ⅙β charts. The quark/quark and lepton/$\overline{\text{lepton}}$ charts can also be produced as sums of the 0*α + ⅙β, 0*α + ½β, and ½α + 0*β charts.

Lepton /$\overline{\text{lepton}}$ crosses and quark /$\overline{\text{quark}}$ crosses

Figure 13, which is easy to remember, makes the data in **Table 2** more transparent, where lepton/anti-lepton duplets are represented by the tips of bidirectional arrows of unit length. The horizontal and vertical coordinates of the arrowheads indicate the values of the weak hyper charge and weak isospin charge of the duplets.

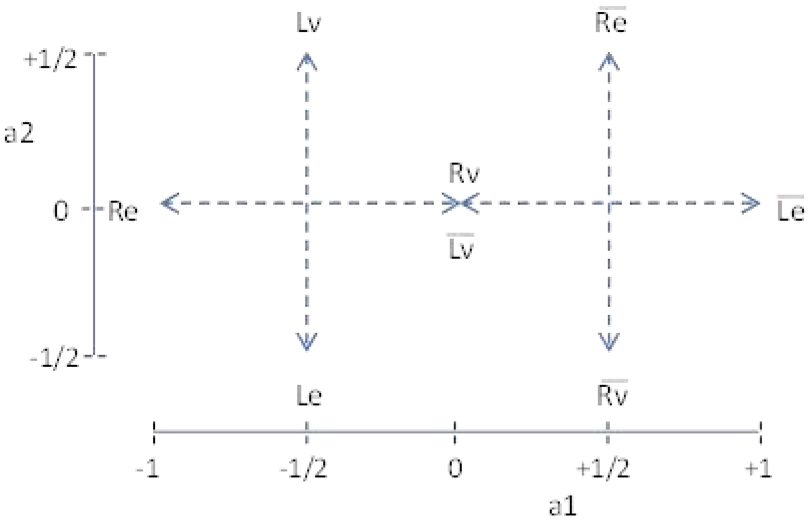


Figure 13. Crosses of lepton duplets. The lepton pairs and $\overline{\text{lepton}}$ pairs are represented by arrow tips of unit length. The horizontal coordinates of the arrow tips correspond to the a1-bits and $\overline{\text{a1}}$-bits, while the vertical coordinates correspond to the a2-bits and $\overline{\text{a2}}$-bits. The cross on the left depicts the lepton duplets, and the cross on the right depicts the $\overline{\text{lepton}}$ duplets.

Without memorizing numbers from the diagram, every lepton/$\overline{\text{lepton}}$ duplet can be recalled along with its (a1, a2) bits. It is sufficient to remember the double cross shape and that the arrows are of unit length, with the origin of the Cartesian coordinate system at the center.

The coordinates of the arrowheads on the left cross match the quantum numbers (a1, a2) of the lepton duplets, while the coordinates of the arrowheads on the right cross match the quantum numbers (a1, a2) of the $\overline{\text{lepton}}$ duplets.

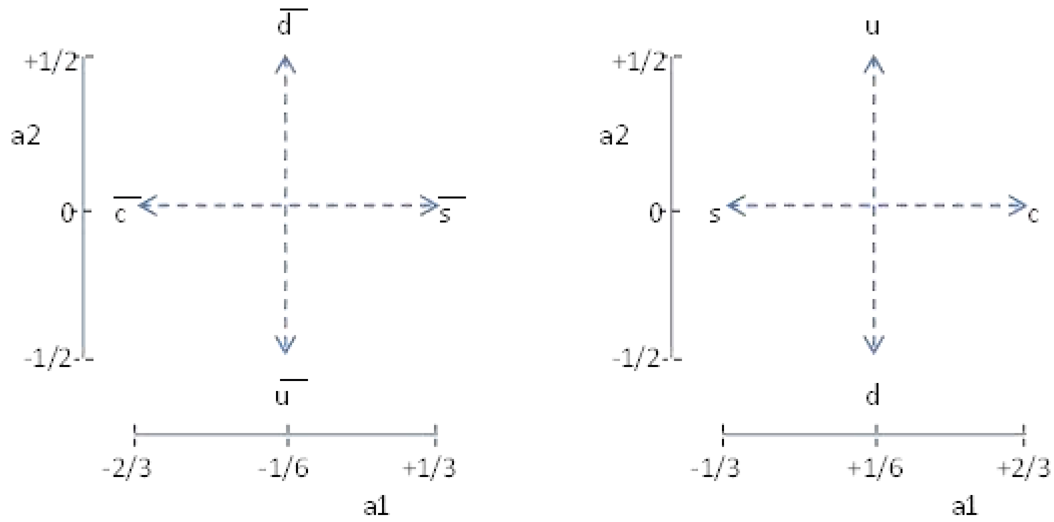


Figure 14. Crosses of $\overline{\text{quark}}$ duplets and quark duplets. The duplets are represented by the tips of arrows of unit length. The horizontal coordinates of the arrowheads correspond to the a1 bits and ā1 bits, while the vertical coordinates of the arrowheads correspond to the a2 bits and ā2 bits. The left cross depicts the anti-quark duplets, and the right cross depicts the quark duplets.

The arrowheads of the two crosses are symmetric for the reflections [#], [├], [ı], [/], [\], [·] and the rotation k* π (k = integer).

The lepton/$\overline{\text{lepton}}$ duplets [–]-symmetrical on the chart are positioned identically in the two crosses (difference in their horizontal coordinates: 1).

The common point of the two crosses is the origin (0, 0), whose two coordinates are the a1-bit and a2-bit of the Rν-duplet and the $\overline{\text{L}}$ν-duplet.

Diametric rotation changes only the a1 values; therefore, the a2 values of quark/$\overline{\text{quark}}$ duplets and their corresponding lepton/$\overline{\text{lepton}}$ duplets are equal, while their a1 values differ by 2/3 and 1/3. We plotted the lepton/$\overline{\text{lepton}}$ crosses **(Figure 13)**, and the quark/$\overline{\text{quark}}$ crosses **(Figure 14)** horizontally shifted by 2/3, and 1/3.

The two crosses of leptons/$\overline{\text{leptons}}$ and the two crosses of quarks/$\overline{\text{quarks}}$ can be derived from a single primordial cross, centered at the origin (0,0), with arrowhead coordinates (a1, a2) of (-1/2,0), (+1/2,0), (0,-1/2), (0,+1/2).

These (a1, a2) number pairs also can be considered as the coordinates of a vector of length 1/2 rotated in a plane, (1/2) * sin(k * π/2), (1/2) * cos(k * π/2), where k = 0, 1, 2, 3, or as complex numbers, where a1 is the real part and a2 is the imaginary part.

The primordial cross with a Δa1 = ±1/2 shift gives the two crosses of leptons and $\overline{\text{leptons}}$, while a Δa1 = ±1/6 shift gives the two crosses of quarks and $\overline{\text{quarks}}$. The shift of the primordial cross adds a value of ±1/2 (for a1-bits of leptons and $\overline{\text{leptons}}$) or ±1/6 (for a1-bits of quarks and $\overline{\text{quarks}}$) to the a1 coordinate of the arrowheads.

Therefore, the fermion and anti-fermion duplets can be expressed as the sum of the coordinates (-1/2, 0), (+1/2, 0), (0, -1/2), (0, +1/2) and a shift term of (±1/2, 0) or (±1/6, 0) **(Table 6)**.

The same is reflected in the α, β coordinate system of the charts, where one of the a1 and a2 bits of the couples always contains a ±1/2 term, and the a2-bit contains another term, which in the case of quarks/$\overline{\text{quarks}}$ is Γ2*β = ±1/6, and in the case of leptons/$\overline{\text{leptons}}$ is Γ2*β = ±1/2.

The derivation of the duplets as a shift of the point (0, 0) reflects the components q = a1 + a2 of the electric charge value of the duplets as well as the further algebraic partition of the a-bits into components a-bit = Γ1*α + Γ2*β.

A

	a1	a2	a1	a2	a1	a2	a1	a2
Arrowheads of prim. cross	-1/2	0	0	-1/2	0	+1/2	+1/2	0
Shift term of lepton duplets	-1/2	0	-1/2	0	-1/2	0	-1/2	0
Σa1, Σa2	-1	0	-1/2	-1/2	-1/2	+1/2	0	0
Lepton duplet	Re		Le		Lv		Rv	

B

	a1	a2	a1	a2	a1	a2	a1	a2
Arrowheads of prim. cross	-1/2	0	0	-1/2	0	+1/2	+1/2	0
Shift term of quark duplets	+1/6	0	+1/6	0	+1/6	0	+1/6	0
Σa1, Σa2	-1/3	0	+1/6	-1/2	+1/6	+1/2	+2/3	0
Quark duplet	strange		down		up		charm	

C

	a1	a2	a1	a2	a1	a2	a1	a2
Arrowheads of prim. cross	-1/2	0	0	-1/2	0	+1/2	+1/2	0
Shift term of $\overline{\text{lepton}}$ duplets	+1/2	0	+1/2	0	+1/2	0	+1/2	0
Σa1, Σa2	0	0	+1/2	-1/2	+1/2	+1/2	+1	0
$\overline{\text{Lepton}}$ duplet	L$\overline{v}$		R$\overline{v}$		R$\overline{e}$		L$\overline{e}$	

D

	a1	a2	a1	a2	a1	a2	a1	a2
Arrowheads of prim. cross	-1/2	0	0	-1/2	0	+1/2	+1/2	0
Shift term of $\overline{\text{quark}}$ duplets	-1/6	0	-1/6	0	-1/6	0	-1/6	0
Σa1, Σa2	-2/3	0	-1/6	-1/2	-1/6	+1/2	+1/3	0
$\overline{\text{Quark}}$ duplet	$\overline{\text{charm}}$		$\overline{\text{up}}$		$\overline{\text{down}}$		$\overline{\text{strange}}$	

Table 6. Derivation of lepton and quark duplets from the coordinates of the primordial cross. Adding a shift term of (±1/6, 0) or (±1/2, 0) to the coordinates of the primordial cross vertices generates the a-bits of the duplets.

Additive structure of the diametric replacement (addition of duplets to anti-duplets)

Considering that the chart is an α, β coordinate system, the differences of the duplets can be easily read from their arrow representation in the chart **(Figure 6)**. The difference of the starting points of the arrows is the difference of the a1 bits, while the difference of the endpoints is the difference of the a2 bits.

At the same time, the difference between two quark duplets corresponds to the sum of a quark duplet and an $\overline{\text{quark}}$ duplet.

A quark and an anti-quark (fermions) form a composite boson particle, a meson. Mesons were modeled on the SD-graph **(1)**. The sum of the a1-bits and a2-bits of a quark duplet and an $\overline{\text{quark}}$ duplet equals to the hyper charge (Y') and isospin (I) of a meson.

The elements of **Table 7/A** represent the Σa1, Σa2 values of quark duplet + $\overline{\text{quark}}$ duplets. Lepton/$\overline{\text{lepton}}$ charts are generated from the quark chart via a diametric rotation (adding ±1/3 or ±2/3); therefore, the values Σa1, Σa2 of a lepton duplet + $\overline{\text{lepton}}$ duplet equal to the values Σa1, Σa2 of the corresponding quark duplet + $\overline{\text{quark}}$ duplet pair **(Table 7/A-B)**.

During the diametric replacement of a0 = 0, due to the indistinguishable starting and ending points of the 0 arc, the order of the a1-bit and a2-bit can be interchanged, leading to the formation of **reverse lepton duplets**, which can be indicated with arrows in the opposite direction on the lepton chart.

A		Quark duplets: d		u		s		c	
		a1	a2	a1	a2	a1	a2	a1	a2
		+⅙	−½	+⅙	+½	−⅓	0	+⅔	0
$\overline{\text{Quark}}$ duplets	a1 a2								
anti-d	−⅙ +½	0	0	0	+1	−½	+½	+½	+½
anti-u	−⅙ −½	0	−1	0	0	−½	−½	+½	−½
anti-s	+⅓ 0	+½	−½	+½	+½	0	0	+1	0
anti-c	−⅔ 0	−½	−½	−½	+½	−1	0	0	0

B		Lepton duplets: Le		Lv		Re		Rv	
		a1	a2	a1	a2	a1	a2	a1	a2
		−½	−½	−½	+½	−1	0	0	0
$\overline{\text{Lepton}}$ duplets	a1 a2								
$\overline{\text{Re}}$	+½ +½	0	0	0	+1	−½	+½	+½	+½
$\overline{\text{Rv}}$	+½ −½	0	−1	0	0	−½	−½	+½	−½
$\overline{\text{Le}}$	+1 0	+½	−½	+½	+½	0	0	+1	0
$\overline{\text{Lv}}$	0 0	−½	−½	−½	+½	−1	0	0	0

Table 7. Σa1, Σa2 values of quark + anti-quark pairs **(A)** and Σa1, Σa2 values of lepton + anti-lepton pairs **(B)**. The (Σa1, Σa2) entry at the intersection of each row and column represents the sum of the a1 and a2 bits of the pair indicated in the headers of the respective row and column. We marked the main diagonal and the secondary diagonal of the tables with dashed lines.

The summation (**Table 7**) results in the lepton duplets (row 4 of the table), the lepton duplets (row 3 of the table), the reverse lepton duplets (row 2 of the table), and the reverse lepton duplets (row 1 of the table). Due to the analogy with particle physics, these duplets are referred to as **meson duplets**.

Symmetries of the additive structure of the diametric replacement (symmetries of meson duplets)

Table 7/A has been arranged so that the column headers in the odd columns are of the down type, and in the even columns are of the up type quark duplets; in columns 1-2 are first-generation quark duplets, and in columns 3-4 are second-generation quark duplets. The row headers correspond to the respective quark duplets.

In **Table 7/B**, the column headers in the odd columns are electron duplets, and in the even columns are neutrino duplets; in columns 1-2 are L lepton duplets, and in columns 3-4 are R lepton duplets. The row header corresponds to the respective lepton duplet.

The rows and columns of the table are referred to as 1/4 segments, consecutive odd and even row pairs and column pairs as 1/2 segments, and four rows and four columns as 1/1 segments.

The reflections of the **Table 7** are denoted by the previously introduced symbols [#], [–], [|], [·], [/], [\].

Geometric reflections can also be replaced by reversing and interchanging the horizontal and vertical coordinate axes of the table (the order of rows and columns). [\] = swapping coordinate axes, [·] = reversing coordinate axes, [/] = swapping and reversing coordinate axes.

Reversing the order of rows and columns can also be interpreted as **swapping segments** of the coordinate axes (**Figure 15**).

Swapping and reversing the 1/2 segments produces the same effect as reversing the 1/1 segment, regardless of the order of the two transformations (**Figure 15**). If the two swaps are performed on both the rows and columns of the table, we obtain [·]-reflection of the table, that is, the table is rotated by ±π.

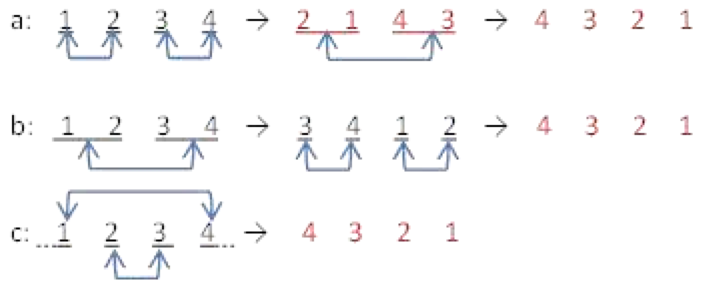


Figure 15. Reversing the 1→2→3→4 sequence by reversing and swapping the 1/2 segments. The underlined number pairs represent the 1/2 segments. The arrows connecting the elements of a segment indicate the reversal of that segment, while the arrows connecting the two segments indicate the swapping of the segments. Circular sequences in the opposite direction are distinguished by black and red colors.
(The circular sequence →1→2→3→4→ can be divided into 1/2 segments in two ways, and swapping the 1/2 segments preserves the traversal order, while reversing the 1/2 segments reverses the traversal direction.)

The geometrical symmetries of **Tables 7/A-B** correspond to the symmetries of meson duplets; the three hierarchical transformations of the table correspond to the interchange of the three characteristics of quark/quark duplets:

1. Exchanging the coordinate axes of the table corresponds to swapping the quark duplets and quark duplets, and reverses the sign of the a-bits in the table. This has the same effect as a [\]-reflection, that is, the elements of the table (a1, a2 pairs) are obliquely symmetric relative to the main diagonal.
2. Exchanging the 1/2 segments corresponds to swapping the generations and interchanges the a1- and a2-bits of the duplets in the table fields. This has the same effect as a [/]-reflection of the table.
3. Reversing the 1/2 segments corresponds to swapping the charges q= ±1/3 and q= ±2/3, and reverses the sign of the a-bits in the table. This has the same effect as a [\]-reflection.

Tables 7/A-B indicate the swapping of rows and columns with colored arrows.

Interchanging the two generations of quark duplets produces the same change in **Table 7/A** as interchanging the L- and R-chirality lepton duplets in **Table 7/B**. Similarly, interchanging quark duplets with q=±1/3 and q=±2/3 induces the same change in the table as swapping the charged and neutral lepton duplets.

Therefore, the designations used to distinguish quark duplets (ordinary/anti, generations, electric charge) and those used to distinguish lepton duplets (ordinary/anti, L/R, neutral/charged) denote the common symmetry properties of the diametric replacement.

The appearance of reverse lepton/$\overline{\text{lep}}$ton duplets is a necessity of the diametric replacement and ensures the symmetry between the differences of lepton duplets **(Table 7/B)** and the differences of quark duplets **(Table 7/A)**.

Subtraction of duplets

The a-bits of a duplet and the corresponding anti-duplet are opposite, so adding an anti-duplet is equivalent to subtracting an ordinary duplet. Consequently, **Table 7/A** contains the differences of quark duplets, while **Table 7/B** contains the differences of lepton duplets.

The crosses of lepton duplets, $\overline{\text{lep}}$ton duplets, quark duplets, and quark duplets are congruent **(Figure 13, Figure 14)**, although they are located differently in the coordinate system. Therefore, the 16 difference vectors between the arrowheads (**Figure 16**) are the same across the four crosses, and the coordinates of the 16 vectors correspond to the duplets in **Table 7/A** and **Table 7/B**.

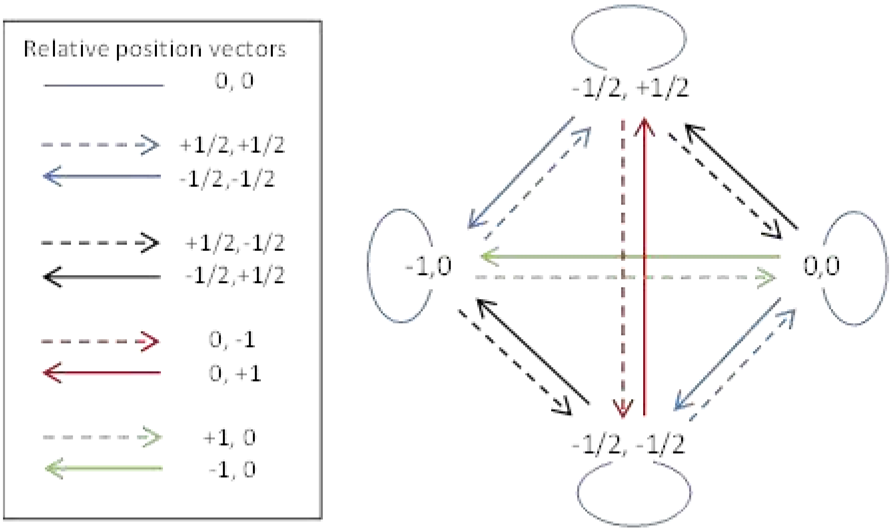


Figure 16. Relative position vectors of lepton pairs. The horizontal and vertical coordinates of the four points represent the (a1, a2) bits of the lepton pairs, as in Figure 12. The positions of each point relative to the other points are depicted and listed. Oval arrows symbolize the zero vector. The coordinates of the anti-lepton pairs form the same system with opposite coordinates but with the same relative position vectors (not shown).

Fermion quads

By reading the charts as α, β coordinate systems, we can verify that the electric charge q = a1 + a2 of the quark duplets represented by colored arrows **(Figure 10)** can be decomposed into a +1/6 and a ±1/2 component. The +1/6 component is always contained in the a1-bit, while the ±1/2 component appears in one of the two a-bits. In lepton duplets, the +1/6 component is replaced by -1/2.

The same can also be seen in tabular form **(Table 4)**.

We illustrated the same by generating the a-bits of the duplets through the translation of the primordial cross (Banach circle with radius 1/2). As a result, the a-bits were decomposed onto the coordinates of the arrowheads (–1/2, 0), (+1/2, 0), (0, –1/2), (0, +1/2) and into a translation term of (±1/2, 0) and (±1/6, 0) **(Figure 13, Figure 14, Table 6)**.

This time, the coordinates of the primordial cross arrowheads (–1/2, 0), (+1/2, 0), (0, –1/2), (0, +1/2) are represented as the configurations of a pin (**I**) placed in a 2D **quad (Figure 17)**. The pin carries an absolute value of 1/2.

The horizontal a1/a2 dimension of the quad indicates whether the pin represents the a1 (a1-bit) or a2 coordinate (a2-bit) of a duplet. The vertical –/+ dimension indicates the sign of the pin.

One pin placed in the quad represents a core **pin configuration** (**fermion/fermion core**) in the pin configuration space of the quad.

The value of a core pin configuration is a pair of numbers (a1, a2), where a1 is the value of the pin in the first column of the quad, and a2 is the value of the pin in the second column of the quad.

The (a1, a2) is called a core fermion duplet. The core quark quads are the [−]-mirror images of quark quads, lepton quads are the [−]-mirrors of lepton quads.

From the core fermion/fermion duplets, the shift term (±1/2, 0) or (±1/6, 0) added to the a1/a1 value, generates the a-bits of the lepton/lepton duplets and the quark/quark duplets. The name 'shift term' comes from the fact that the crosses of the fermion/fermion duplets were derived by shifting the primordial cross by (±1/2, 0) or (±1/6, 0). In a suitable coordinate system, the difference between the two shift terms (½ – 1/6 = 1/3) can also be interpreted as a baryon number, which distinguishes quark/quark duplets from lepton/lepton duplets.

Considering the α, β coordinate system of the a-bits or the horizontal shift of the primordial cross makes it clear that a nonzero contribution of Γ2 *β = ±1/6 or Γ2 *β = ±1/2 is contained only in the a1/a1-bits, and this contribution is an essential part of every a1/a1-bit.

The shift term is represented by a 1/6-value pin (blue) placed in the a1 column of the quark/quark quads and the 1/2-value pin (red) placed in the a1 column of the lepton/lepton quads.

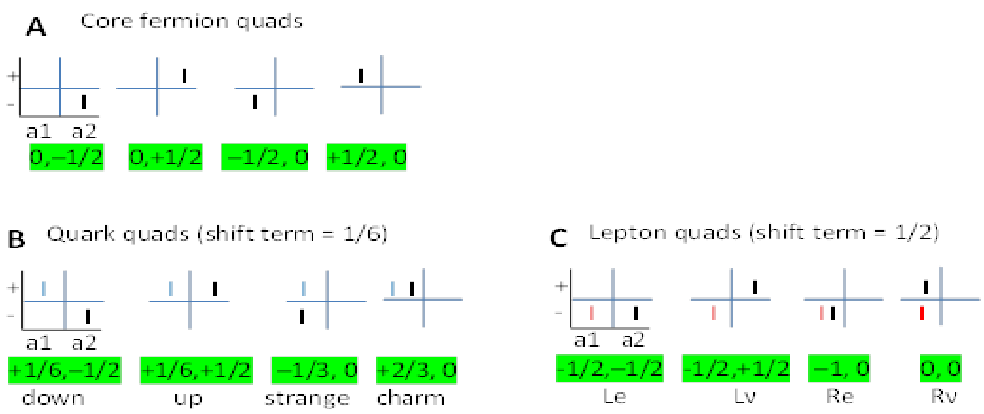


Figure 17. The fermion/fermion core diametric pin configurations (quads) **(A)**. The quark quads **(B)** and the lepton quads **(C)**. The dimensions of the quads are labeled on the first quad. The values of the configurations are highlighted in green.

The lepton duplets are of the form (core a1 – 1/2, a2), the lepton duplets (core a1 + 1/2, a2), the quark duplets (core a1 – 1/6, a2), and the quark duplets (core a1 + 1/6, a2). These constitute the row and column headers of Tables 8/A, B.

The system of 4 quark duplets and lepton duplets, expressed as a pin configuration, is identical in other way; the only difference lies in the value of the shift term.

The lepton/$\overline{\text{lepton}}$ duplets were produced from the quark/quark duplets by Dturn = +1/3 and Dturn = -2/3 **(Figure 10)**. The quad form also reflects that the D_{turn} transformation only means swapping the shift terms, while the fermion/fermion core remains unchanged.

D_{turn} = +1/3 swaps the quark shift term (+1/6) to the lepton shift term (+1/2), and the quark shift term (-1/6) to the lepton shift term (-1/2). D_{turn} = -2/3 swaps the quark shift member (+1/6) to the lepton shift member (-1/2), and the quark shift member (-1/6) to the lepton shift member (+1/2).

Due to the identity of the core quads, the table of quantum number sums for quark+ anti-quark and lepton+anti-lepton **(Table 8/A, B)** is identical. The D_{turn} = +1/3 and D_{turn} = -2/3 only change the shift term, but they do not alter the quark/anti-quark core quad. Therefore, the table of quantum number sums remains identical even if the quarks/anti-quarks in the row and column headers are replaced by leptons/anti-leptons generated by D_{turn} = +1/3 or D_{turn} = -2/3.

We can distinguish between these two types of lepton+ $\overline{\text{lepton}}$ tables by assigning orientations to the pins such that in the fermion core quads the spin is in the upward (↑) direction, whereas in the fermion core quads it is in the opposite (↓) direction **(Table 8/A, B, C)**. As a result, in the two types of lepton+ lepton tables, the values of the quads remain identical, but the spin directions are opposite.

This is illustrated in **Table 8**, where the quark/anti-quark rows and column headers **(Table 8/A)** have been replaced by leptons/$\overline{\text{leptons}}$ generated by D_{turn} = -2/3 **(Table 8/B)** and by leptons/leptons generated by D_{turn} = +1/3 **(Table 8/C)**.

The shift term of the fermion and fermion quads has equal magnitude and opposite sign, thus they neutralize each other, which is why they are not shown in the result of the summation.

The configuration space of the anti parallel pin pairs

We rewrite the sum of fermion duplet + fermion duplet pairs **(Table 7)**, this time representing the duplets as quads. The pins of the fermion and fermion quads are in opposite directions, therefore their sum forms an anti parallel pin pair. The two shift terms with identical a1 coordinates but opposite direction and opposite sign cancel each other out as a result of the addition, so they have not been shown.

The pins of the fermion and the anti-fermion quads are in opposite directions, therefore their sum forms an anti parallel pin pair. The anti parallel (↓↑) pin pair has 16 configurations in the configuration space represented by the quads.

The sums (lepton quad + anti-lepton quad) **(Table 8/B)** as well as the sums (quark quad + anti-quark quad) **(Table 8/A)** are equal to the quad values of the 16 distinct configurations of anti parallel (↓↑) pin pairs. The quad values of these 16 different configurations correspond to the hyper charge and isospin (Y', I) quantum numbers of mesons, in accordance with the fact that meson particles are quark–anti-quark pairs.

The summation of lepton + anti-lepton quads yielded the same meson quantum numbers, even though lepton + anti-lepton pairs do not actually form mesons or other composite particles. Quark/quark duplets, corresponding to physical particles, can be given ±1/3 baryon charge, which distinguishes them from lepton/lepton duplets. The baryon charge was previously modeled on the SD graph **(1)**.

A	down +1/6, -1/2	up +1/6, +1/2	strange -1/3, 0	charm +2/3, 0
$\overline{\text{down}}$ -1/6, +1/2	0, 0	0, +1	-1/2, +1/2	+1/2, +1/2
$\overline{\text{up}}$ -1/6, -1/2	0, -1	0, 0	-1/2, -1/2	+1/2, -1/2
$\overline{\text{stran.}}$ +1/3, 0	+1/2, -1/2	+1/2, +1/2	0, 0	+1, 0
$\overline{\text{charm}}$ -2/3, 0	-1/2, -1/2	-1/2, +1/2	-1, 0	0, 0

B	Le -1/2, -1/2	Lv -1/2, +1/2	Re -1, 0	Rv 0, 0
$\overline{\text{Re}}$ +1/2, +1/2	0, 0	0, +1	-1/2, +1/2	+1/2, +1/2
$\overline{\text{Rv}}$ +1/2, -1/2	0, -1	0, 0	-1/2, -1/2	+1/2, -1/2
$\overline{\text{Le}}$ +1, 0	+1/2, -1/2	+1/2, +1/2	0, 0	+1, 0
$\overline{\text{Lv}}$ 0, 0	-1/2, -1/2	-1/2, +1/2	-1, 0	0, 0

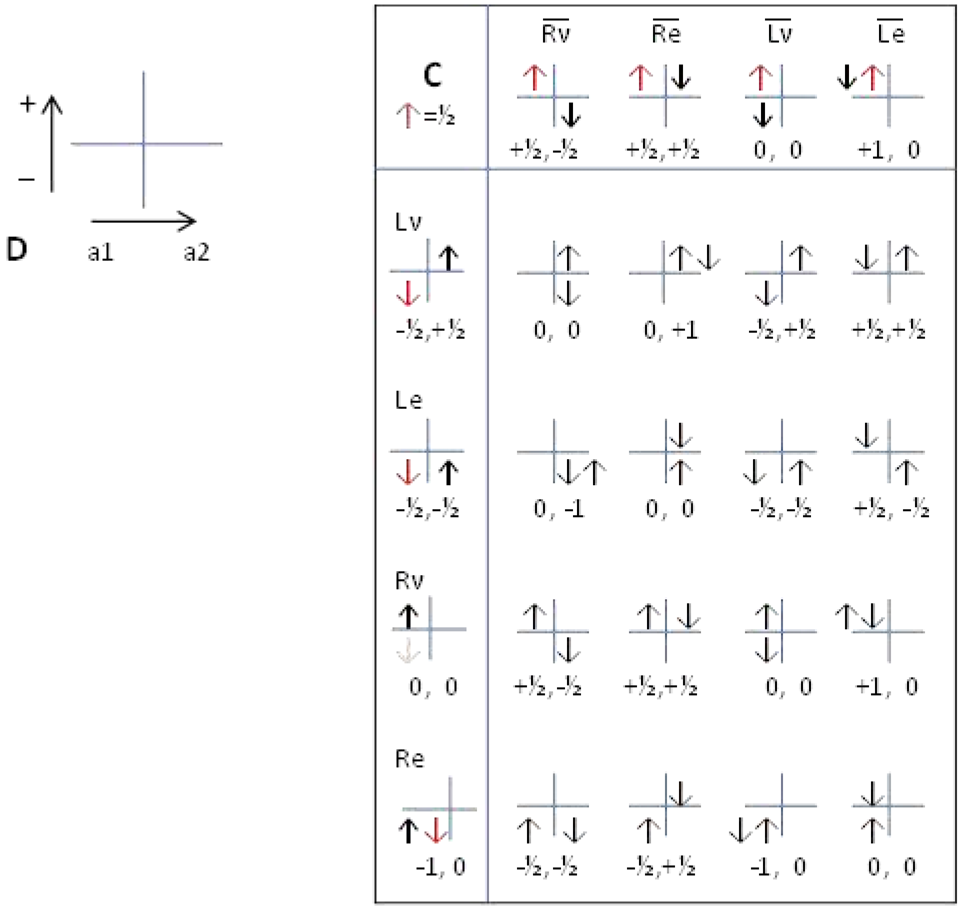


Table 8. Summing the values of quads representing fermion and fermion duplets (row and column headers). In the headers, there are quark/quark quads **(A)**. Instead of quark/quark quads in the headers, there are lepton/lepton quads generated by D_{turn} = -2/3 **(B)**. Instead of quark/quark quads in the headers, there are lepton/lepton quads generated by D_{turn} = +1/3 **(C)**. The quad at the row-column intersection is the sum of the quads in the row and column headers. The number pair below the quad is its value. The a1/a2 and –/+ dimensions of the quads **(D)**.

Symmetries of the diametric pin configuration space

The quad representation allows for the illustration of the symmetries between the properties (a1/a2 and –/+) of the fermion/fermion couples' a-bits (quantum numbers). The reflections of the positions of the pins within the quads and the reflections of the quads within the table are denoted by the previously introduced symbols [–], [|], [·], [/], [\], while the inversion of the pins is indicated by ↨ **(Table 9)**. A reflection in the quads is indicated with a lower index q (quad), and a reflection in the table with a lower index t (table). Two transformations are considered equivalent if they transform the (i, j) quad of the table into the same quad, regardless of the specific (i, j) case. Equivalent transformations are marked with a sign “=” **(Table 9)**. A quad generated by inverting the pins (↨) is called the opposite of the quad, and two transformations are considered to have opposite effects if they transform each (i, j) quad into its opposite.

As in previous cases, here we also apply sequence segmentation and the reversal of the order of segments.

The table demonstrates that the $[–]_q$ and $[\backslash]_t$ transformations, as well as the $[|]_q$ and $[/]_t$ transformations, are pairs of opposite transformations. Furthermore, a $[\cdot]_t$-reflection of the table is equivalent to performing a simultaneous $[\cdot]_q$-reflection in each quad. A [·]-reflection corresponds to a rotation of $\pm\pi$ around the center, so rotating the table by $\pm\pi$ is equivalent to rotating the pins by $\pm\pi$ in each quad. The rotations are interpreted such that rotation within a quad changes the positions of the pins within the quad's coordinate system while preserving the ↓,↑ states. Rotation around the center of the table changes the position of the quad within the table, without changing the position and direction of the pins inside the rotated quads.

Quad Projection		Table Projection		Table Rearrangement
$[-]_q$ (= - ↔ +)	=	$[\backslash]_t$ * ↕	=	reverse of 1/2 segments
$[-]_q$ * ↕	=	$[\backslash]_t$	=	reverse of 1/2 segments * ↕
$[\mathrm{I}]_q$ (= a1 ↔ a2)	=	$[/]_t$ * ↕	=	change of 1/2 segments
$[\mathrm{I}]_q$ * ↕	=	$[/]_t$	=	change of 1/2 segments * ↕
$[-]_q * [\mathrm{I}]_q = [\backslash]_q * [/]_q$	=	$[-]_t * [\mathrm{I}]_t = [\backslash]_t * [/]_t$	=	change of reversed 1/2 segments

Table 9. Quad and table transformations with the same effect in the configuration space of (↓↑) pin pairs. The meaning of the character symbols is explained in the text. Reverse of 1/2 segments: reversing the direction of the segments, i.e., swapping the order of the 1/4 segments. Change of 1/2 segments: swapping the two 1/2 segments.

Meson-crosses

The a-bits of the 16 meson pairs correspond to the a-bits of the 4 lepton pairs, the 4 lepton pairs, and the 8 reversed pairs containing the swapped a1 and a2 bits **(Table 7)**.

Swapping the coordinates means a [/]-reflection, the a1 and a2 bits of the 16 meson pairs represent the coordinates of the tips of a double cross and a double cross rotated by π (**Figure 18**). It is symmetric under the reflections [#],[–], [I], [/], [\], [·] and under the rotation k* π/2 (k = integer).

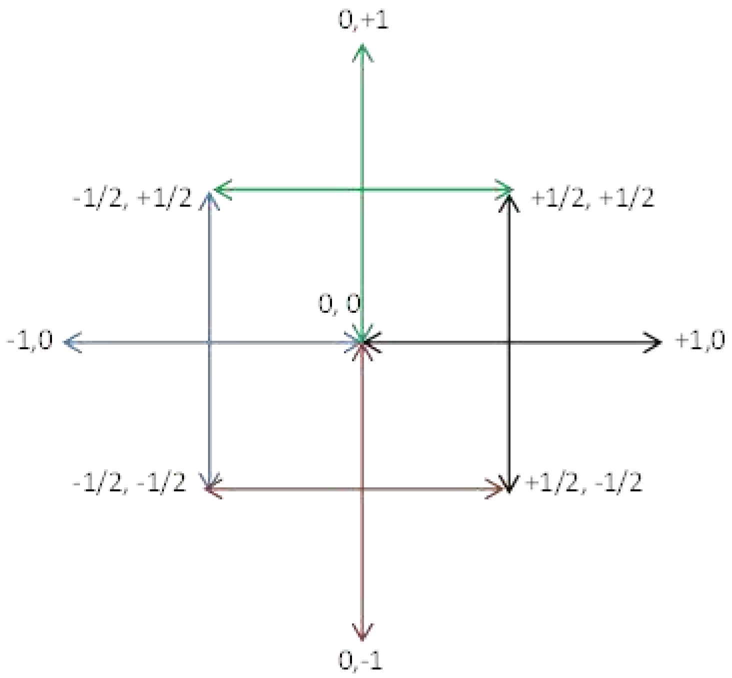


Figure 18. The coordinates of the eight endpoints of a double cross and a double cross rotated by π/2 (a1 and a2 values of the meson duplets).

The rotated double cross can also be derived from the arrowheads of the primordial cross by adding ±1/2 to the a2 value of the (a1, a2) coordinates (data not shown). Each arrowhead of the two double crosses can be generated by taking every combinations of two steps of length ±1/2 along the coordinate axes from the origin. The arrowheads form a Banach circle of radius r = 2 units. In Banach geometry, the distance between two points on a square lattice is the sum of the differences of their coordinates, so it is the shortest lattice path between two points is measured by the number of lattice units along the lattice lines.

Construction of a meson polyhedron

Several arrowheads of the cross of the meson duplets **(Figure 18)** are indistinguishable because their planar coordinates are identical. The coinciding arrowheads can be separated by assigning them a value in a third, (z) dimension.

To mark the coordinates of the overlapping n arrowheads, we follow a procedure similar to the one that created the arrowheads in two steps. This was done by taking two 1/2-length steps along the arms of the tetra-cross in all possible directions, starting from the point 0,0.

We can also generate each arrowhead by starting from the four peripheral points of the tetra-cross and taking two 1/2-length steps in all possible directions along the arms of the cross.

The steps are radial with a length of 1/2 or perpendicular to these. The radial distance of the arrowhead is the number of 1/2-length radial steps along the shortest path from the periphery to the arrowhead. The radial distance of the arrowhead is 0 for the four peripheral arrowheads, 1 for the eight intermediate-position arrowheads, and 2 for the four central arrowheads.

The z-axis reflects how 0, 1, or 2 steps can be realized.

A 0 step can be realized in only one way; therefore, the z-coordinate of the four isolated, peripheral arrowheads is 0.

One step can be taken in two ways, with the value of z being -1/2 and +1/2. The eight intermediate position arrowheads are paired, and z = -1/2 and z = +1/2 distinguish the members of each pair.

Two steps can be taken in four ways: (-1/2, -1/2), (-1/2, +1/2), (+1/2, -1/2), (+1/2, +1/2), hence the value of z can be -1, 0, 0, +1. These values are the z coordinates of the four central arrowheads.

Accordingly, in the 3rd dimension, we have given the value 0 to the single arrowheads, the values -1/2 and +1/2 to two coincident arrowheads, and the values -1, 0, 0, +1 to four coincident arrowheads.

As a result, we obtained the vertices and the center of a 3D (a1, a2, z) polyhedron **(Figure 19)**. Each vertex of the polyhedron corresponds to an arrowhead of the quadrilateral, and the center has two arrowheads with coordinates (0, 0).

The coordinates of each vertex (a1, a2) of the quadrilateral were obtained by summing a quark duplet and a quark duplet **(Table 2 and Table 7)**, therefore the polyhedron constructed from the quadrilateral is called a meson polyhedron.

The projections of the meson polyhedron in the (a1, a2, z) directions on the three perpendicular planes of space are identical and coincide with the meson quadrilateral.

In such a representation, the data in **Table 7** are the coordinates of the vertices of a 24-faced solid covered by congruent triangles of sides √3/2, √3/2, and 1 **(Figure 19/B)** projected onto the plane. Eight arrowheads are located on a spherical shell of radius √3/2, six arrowheads are located on a concentric spherical shell of radius 1, and two arrowheads are located at the center.

The arrowheads can also be imagined as points at the vertices of three congruent, perpendicular hexagons and at their common center, which are popular shapes in literary meson representation.

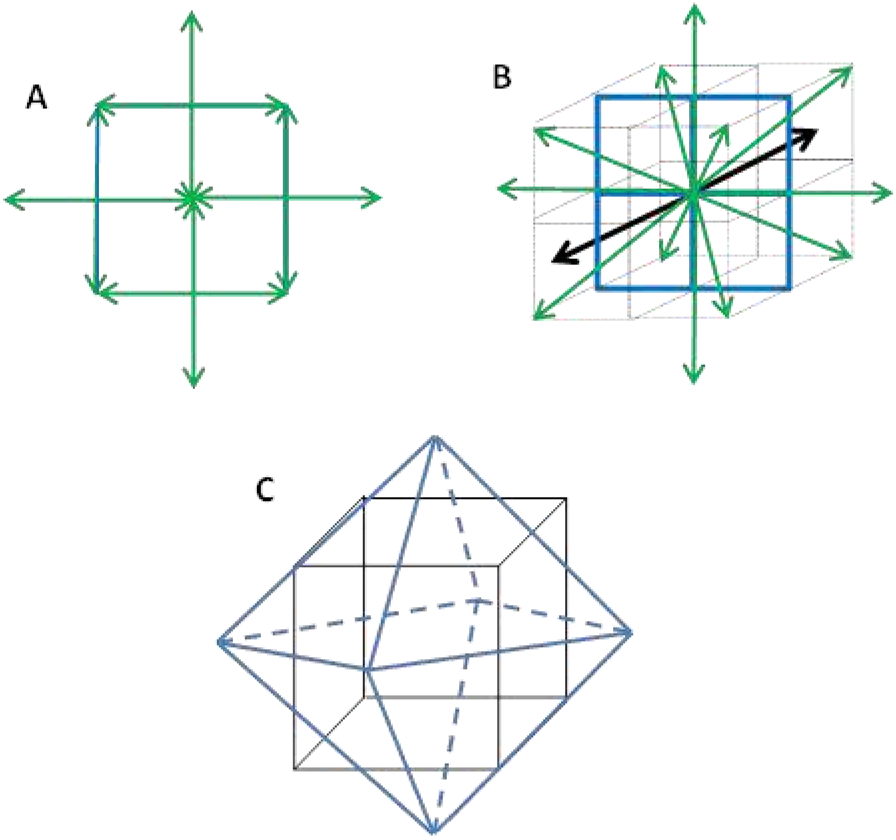


Figure 19. Construction of a meson polyhedron. Tetra-cross of lepton/anti-lepton duplets and reverse lepton/anti-lepton duplets (A) same as in **(Figure 18)**. The blue square is for orientation only. The building blocks of the structure (B) are 1/2 x 1/2 x 1/2 unit cubes. The length of the vertical, horizontal and the perpendicular double-headed arrows is 1, the others are √3/2. Unite cube (black) and its dual octahedron (blue) **(C)**.

Two arrowheads do not separate from each other, which encourages us to consider the vertices of the meson polyhedron as the vertices of two polyhedra, a cube and an octahedron, whose common center is the point 0,0. The coordinates of the vertices of the octahedron correspond to the sum of quark duplets and quark duplets of the same generation, and the vertices of the cube correspond to the sum of pairs of quark duplets and quark duplets of the same generation.

The meson polyhedron is the union of two dual Platonic solids, a cube and an octahedron **(Figure 19/C)**, as if it had stepped out of Johannes Kepler's work *Harmonice Mundi*.

Perhaps Kepler did not hear the whisper of inspiration clearly; could the illustration of the two dual polytopes have accidentally ended up among the laws describing the motion of the planets in his groundbreaking work?

Plato believed thousands of years ago that a few perfect bodies were the building blocks of the world, and it is an unsolved mystery how he became convinced that these bodies were the basic units of the classical elements that built the world.
Pondering his motives, it is also a question of where the still-influential stream of ideas of ancient Greek and Indian atomism, in which Plato's geometric constructions gained meaning, originated.
The sources of the flow of human thought fade into the mists of time, but now we unexpectedly find ourselves again at the beginnings: the vertices of the meson polyhedron are the vertices of two dual Platonic solids, and they actually represent subatomic parts.

Parity tetrahedron of the cube

The vertices of the cube can be decomposed into two other Platonic solids, vertex sets of two congruent regular tetrahedra (**vertex sets S1 and S2**), which are symmetrical about the center of the cube and are marked with filled and empty circles **(Figure 20)**. The tetrahedra are dual of themselves.
The vertices of S1 and S2 have opposite parity, the number of 1s in their 3D coordinates is odd vs. even, so the vertices of the same vertex set differ in an even number of dimensions.
S1 and S2 are invariant against a rotation of any face of the cube by k*π. The projection of the vertices onto any face of the cube projects one vertex of odd and one vertex of even parity onto each other.
The vertices of the same vertex set S are connected by the diagonals of the 2D faces, which are the edges of the tetrahedron (a complete graph with four vertices) **(Figure 20/B, C)**.
Each edge and each 3D diagonal of the cube connects two vertices belonging to the opposite vertex set S **(Figure 20/A)**. Each vertex of S1 is connected to every vertex of the opposite vertex set S2 by 3 edges of the cube and a 3D diagonal.

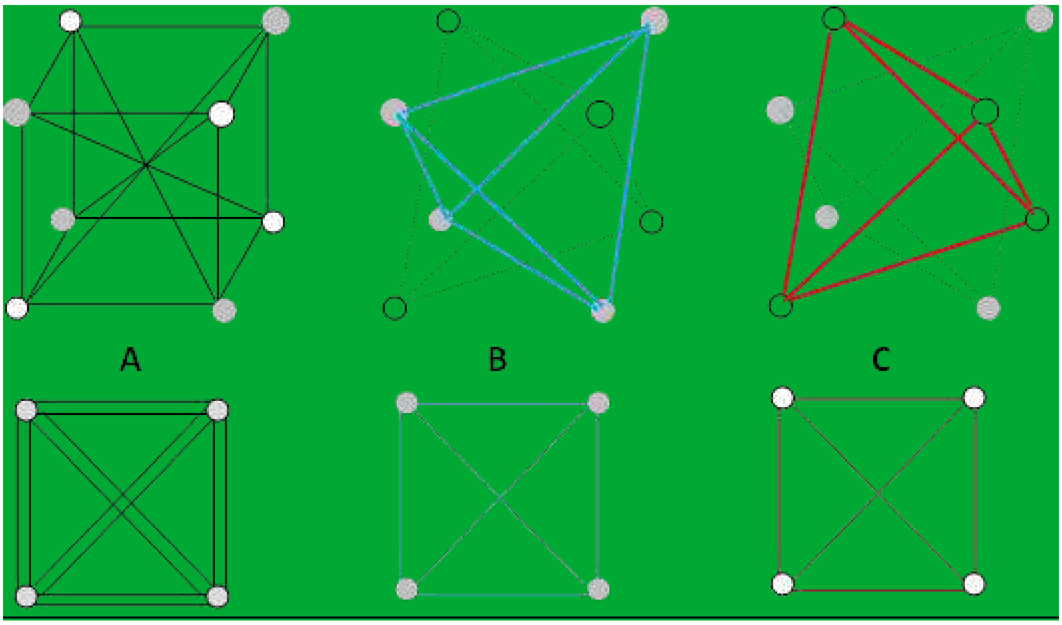
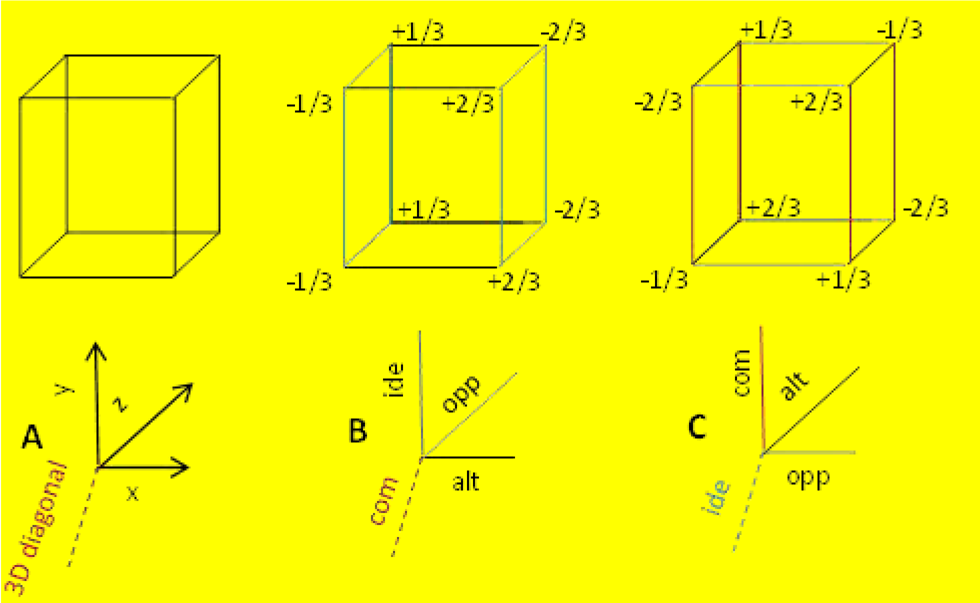


Figure 20. Two disjunct, congruent sets (S1 and S2) of the cube's vertices (open/filled) **(A)**. The tetrahedron (blue) of S1 vertices **(B)**, The tetrahedron (red) of S2 vertices **(C)**. Their projection to an arbitrary face of the cube (below).

A cube can mean many things. It can also represent a four-point hyper graph, where the vertices S1 are the vertices of the graph and the vertices S2 are the hyper edges of the graph. The edges and 3D diagonals of the cube are the degrees of the graph. The vertices S1 and S2 of the cube can be mapped to the rows and columns of a 4 x 4 table, and the edges running from S1 to S2 can also be mapped to the (i, j) data of the table.

The C.I.A.O. cuboids

The diametric replacements of the arc a0 = 0 were created by +1/3, -1/3, +2/3, -2/3 diametric rotations from the diametric replacements of the arc a0 = -1/3.
Each of two alternative rotations –1/3 and +2/3 was applied to the quark duplets, and alternative rotations +1/3 and –2/3 were applied to the quark duplets **(Table 2)**.
Each of two complementer negative rotations (–1/3 and–2/3) created lepton duplets, each of two complementer positive rotations (+1/3 and +2/3) created lepton duplets.
Each of two opposite rotations (same values and opposite signs) applied to a pair of quark/quark duplet resulted in particle/anti-particle duplets with opposite chirality, so both Σa1 and Σa2 are 0.
Each of two alternative rotations (-1/3 and +2/3 or +1/3 and -2/3) produce a lepton duplet and a lepton duplet that are opposite in their three characteristics (particle/anti-particle, charged/neutral, L/R).
The relational structure of the C.I.A.O. relations is in some ways similar to the additive structure of the squares of distances that exists between the vertices of a general cuboid. In a cuboid, regardless of the lengths of the x, y, z edges, the lengths of the parallel edges (and therefore their squares) are equal, the square of the 2D diagonals of the faces is the sum of the squares of the two edges, the 3D diagonals are equal and their lengths are: $x^2 + y^2 + z^2 = t^2$.
The same applies to a cuboid where the square of the x, y, z edges is replaced by three C.I.A.O. relations (uni-variate operations). As a result, the square of the 3D diagonal corresponds to the fourth relation. We present two examples where the relationships can be verified **(Figure 21)**.
The similarity between the Pythagorean and C.I.A.O. relations can be seen at a deeper level than concrete numerical values, just as the lengths of the sides of a rectangle are irrelevant when we establish that the sum of the squares of the two perpendicular edges is equal to the square of the diagonal.



Pithagorean (A)	C.I.A.O. (B)	C.I.A.O. (C)
x^2	alt	opp
y^2	ide	com
z^2	opp	alt
$x^2 + y^2$	alt * ide = alt	opp * com = alt
$x^2 + z^2$	alt * opp = com	opp * alt = com
$y^2 + z^2$	ide * opp = opp	com * alt = opp
$x^2 + y^2 + x^2$	alt * ide * opp = com	opp * com * alt = ide

Figure 21. Opp, com, alt, and ide relations corresponded to the edges of a cuboid (A). The correspondences are shown by the coordinate system below the cuboid. The values written at the vertices (B, C) of the cuboid are in the relation indicated in the coordinate system. The relations between the vertices are also distinguished by the color of the edges: alt: black, opp: green, ide: blue, com: red. The dashed line symbolizes the relation corresponding to the 3D diagonals.

The similarity of the Euclidean and C.I.A.O. relations is shown by dual sentences **(Table 10)**. If we interchange the units of the same color of the sentences of the same ordinal number of the two parallel formulations, we obtain statements valid in the other system. The "square of the distance" corresponds to the "relation", and the edges x^2, y^2, x^2 correspond to the relations R1, R2, R3.

The addition of the squares is replaced by multiplication between the relations.

Table 10: Some dual statements valid for cuboids in the Euclidean and the C.I.A.O. system.

Pythagorean statements for a cuboid	C.I.A.O. statements for a cuboid
1, The squares of the distances of the endpoints on parallel edges and diagonals is equal.	1, The relation of the endpoints on parallel edges and diagonals is equal.
2, The squares of the distances of the endpoints on perpendicular edges are x^2, y^2, x^2.	2, The relation of the endpoints on perpendicular edges is R1, R2, R3.
3, The squares of the distances of the endpoints on the diagonals of parallel planes are equal.	3, The relation of the endpoints on the diagonals of parallel planes is equal.
4, The squares of the distances of the endpoints on the diagonals of parallel planes are $x^2 + y^2$, $x^2 + z^2$, $y^2 + x^2$.	4, The relation of the endpoints on the diagonals of parallel planes is R1*R2, R1*R3, R2 *R3.
5, The squares of the distances of the endpoints on the 3D diagonals are equal.	5, The relation of the endpoints on the 3D diagonals is equal.
6, The squares of the distances of the endpoints on the 3D diagonals are $x^2 + y^2 + x^2$.	6, The relation of the endpoints on the 3D diagonal is R1*R2*R3.

These relationships hold for every initial value of x placed at the origin ($-1 \le x \le +1$), regardless of which three C.I.A.O. relations we use to represent the three edges.

The alt, opp, com, and ide relations are the relations that exist between the vertices of the cuboid and the vertices belonging to the opposite S vertex set, thus having opposite parity, and remain valid regardless of which vertex of the C.I.A.O. coordinate system is chosen as the origin.

Swapping the vertices of any two parallel faces of the rectangular cuboid still preserves all three relations. However, the similarity between the two systems is limited. The C.I.A.O. relations imply certain connections that do not hold on a Euclidean cuboid. As can be seen in the examples presented **(Figure 21/B, C)**, the consequences of the relations include the equality of some edges and diagonals that cannot be equal on a Euclidean cuboid.

The values of the electric charge ($q = a1 + a2$) of the lepton/lepton duplets and the quark/quark duplets form a C.I.A.O. system. At the vertices of the two C.I.A.O. cuboids are the lepton/lepton duplets and the quark/quark duplets **(Figure 22)**, reflecting the correspondence created by rotating the quark chart Dturn = +1/3 and the lepton chart Dturn = -1/3 **(Table 2)**. In the two cuboids, the duplets produced from each other by diametric rotation occupy the same geometric positions, so the difference in charges is ±1/3.

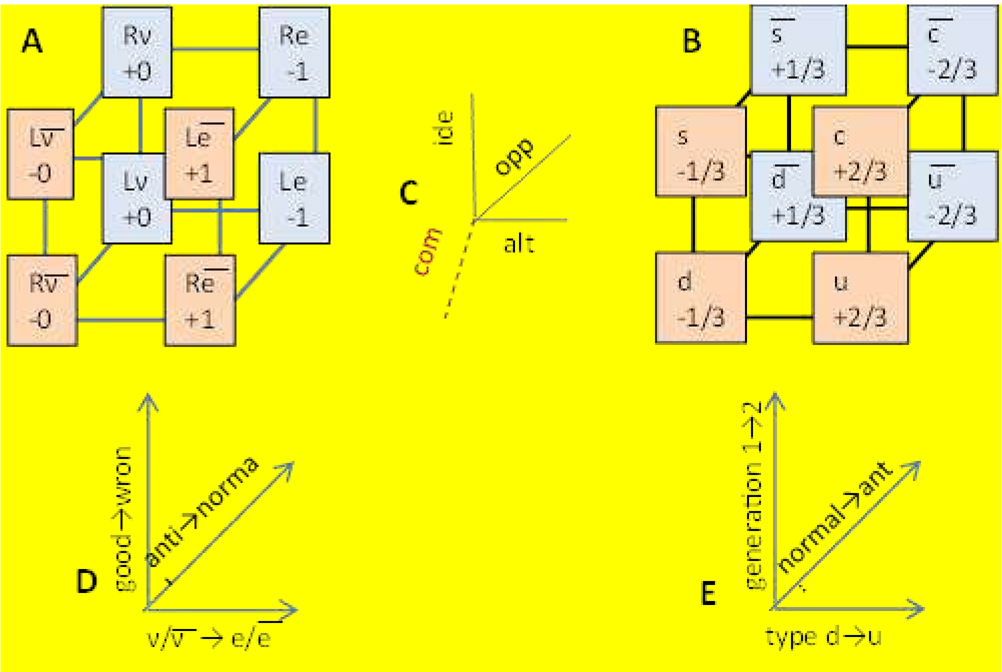


Figure 22. Grouping of lepton/lepton duplets **(A)** and quark /quark duplets **(B)** at the vertices of two cubes by ide, opp, alt, com relations. The electric charges of the duplets are related to each other as indicated in the coordinate system **(C)**. The 3D diagonals correspond to the com relation.
The properties of the lepton/lepton duplets in the ide, opp, alt relations correspond to those indicated in the coordinate system **D**, while the properties of the quark/quark duplets in the ide, opp, alt relations correspond to those indicated in the **E** coordinate system. At the two ends of the 3D diagonals, the duplets differ in all three properties.
On the two cubes, the difference in charge of duplets occupying the same position is ±1/3.

The alt, opp, and ide relations correspond to properties commonly used to characterize particles, which we also borrowed for the classification of duplets **(Figure 22/D, E)**. The alt relation divides lepton/lepton duplets into $v/\bar{v}$ and $e/\bar{e}$ groups, and quark/quark duplets into d-type and u-type. The opp relation designates the groups of regular duplets and anti-duplets. The ide relation distinguishes lepton/lepton duplets with good and wrong handedness, as well as the two generations of quark/quark duplets.

The electric charges of the different categories of leptons are related in various C.I.A.O relations with q = 0 **(Figure 22/A)**. Therefore, if we perform the C.I.A.O operations at the value q = 0, the system of charges of the lepton duplets is generated. Moreover, if we apply the lepton-categorizing relations at the value q = -1/3, we obtain the usual grouping of quarks **(Figure 22/B)**.

The electric charges of lepton/$\overline{\text{lepton}}$ and quark/quark duplets form corresponding relational structures. Using the R operations corresponding to the relations, one can navigate between the electric charges of duplets with different properties. Adding ±1/3 provides the possibility of transition between the appropriate duplets and anti-duplets of the two systems, which ensures their 4D unity. One can switch from a duplet in one system to the corresponding anti-duplet in the other system by adding +1/3, and in the opposite direction, by adding -1/3.

C.I.A.O. relations are symmetric, therefore swapping parallel planes exchanges the duplets in the relation, but does not change the relations indicated on the coordinate axes. Swapping the members of the pairs in the 'ide' relation preserves not only the relations of the cuboid but also the values of the charge at the vertices.

In the quark cuboid, by simultaneously exchanging the members of pairs in the ide, alt, opp relations across all three relations, we obtain the cuboid of opposite chirality **(Figure 23)**. The difference is ±2/3, between the identical positions of the two cuboids. From the charges of the quark duplets, -2/3 produces the charges of the lepton duplets of the other cuboid, and +2/3

produces the lepton charges from the charges of the quark duplets.
Therefore, this interchange of the partners in the relations establishes a correlation between the identical positions of the two cuboids similar to $D_{turn} = \pm 2/3$ instead of the $D_{turn} = \pm 1/3$ relationship.
In this case, however, the change of $\Delta q = \pm 2/3$ is caused by the combined change of a1 and a2, which represents a lepton/lepton duplet with opposite handedness.

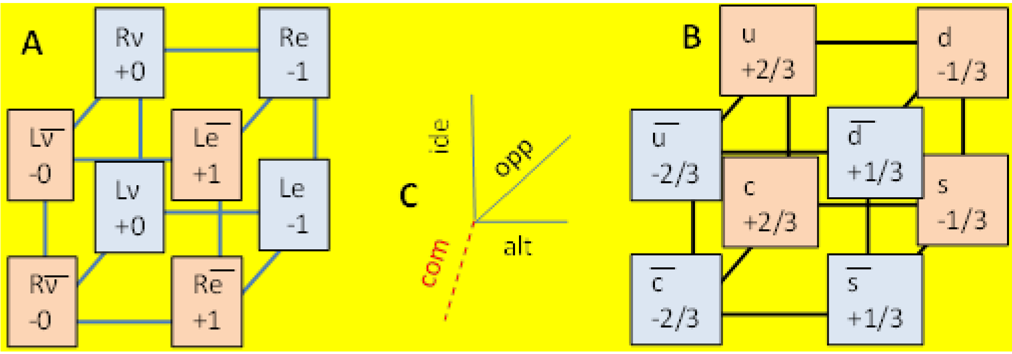


Figure 23. After the central inversion of the vertices of cube **B in Figure 22**, the difference in the charge of the duplets in the same positions of the two cubes is ±2/3.

Let us consider the prehistory of cuboids, that is, the fact that we assigned dimensions to the sums formed from the members of the series -2/3, -1/2, -1/3, -1/6, 0, +1/6, +1/3, +1/2, +2/3 **(Figure 6/A)**.
The members of the series were the a1 and a2 arcs produced by a diametric replacement of an a0 arc, and we established the same relations between the sums formed from them (which we call the electric charges) as those that exist between the arcs connecting two points on a circle.
This also means that these sums, consisting of two arcs, are equal to the arcs connecting two points on a circle.
The relations are valid for the sums because every point of a circle is the endpoint of two alternative arcs, and the sum of the coordinate of an arc-endpoint and ± 1/2 equals the coordinates of the diametrically opposite point in the + and – directions. Therefore, in addition to the fact that the a1 + a2 tandem arc leads to the diametric point of a1, the sum of the angles of a1 and a2 is also equal to the angle of the arc leading to the diametric point of a1.

This equality also appears in the q charge of lepton/lepton duplets with the same q charge in their R and L-handed variants. In the R variants, a1 can take two alternative values, whereas the L variants take their diametric values, and in the R variants q = a1 (with a2 = 0), while in the L variants q = a1 + a2, where a2 = ± 1/2.

The 4D q-cube

The value of the electric charge of fermion/fermion duplets can also be considered as the result of consecutively performed operations. Thus, the common additive structure of the electric charges of lepton/lepton and quark/quark duplets can be mapped onto the vertices of a 4D abstract cuboid, where the edges represent the differences in charges. This can be verified by increasing the number of dimensions from a single point and expanding the abstract extent of the cuboid to 4D **(Figure 24)**. As a result of this process, the electric charge q = a1 + a2 of the fermion/fermion duplets placed at the vertices of the 4D cuboid corresponds to the sum of the four coordinates of the vertices.

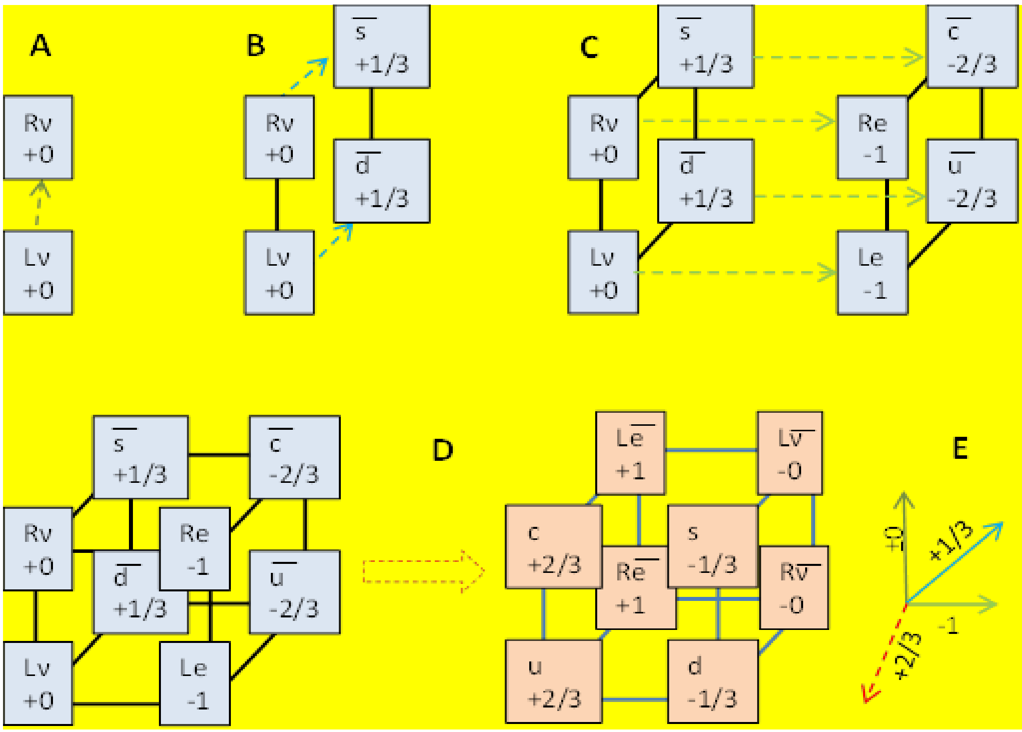


Figure 24. Generating the 4D system of fermion/fermion duplets by increasing the number of dimensions. At each step **(A-B-C-D)** of repeated doubling, the system of fermion/fermion duplets shown at the vertices gains another dimension.
Extension in the fourth-dimension is represented by dashed block arrow **(D)** and dashed line of the coordinate system **(E).**

In four dimensions, the cuboid's abstract extensions are ±0*1/3, ±1*1/3, ±2*1/3, ±3*1/3. The ± signs depend on which duplet we start with and the order in which we expand the dimensions. For example, starting from the Le or Re duplet, expanding in the order 0, +1/3, +2/3, +1 creates the 4D q cuboid. The same values appear in both S vertex sets of the 3D subspaces. In the S vertex sets of disjunct 3D subspace pairs, the q values have opposite signs.

The fourth-dimensional extension is represented by the inner/outer placement of the 3D cuboid **(Figure 24)**.
The lepton, lepton, quark, quark duplets are contained separately on the two disjunct 3D cubes (black and blue) with parallel faces at a +1/3 distance.
Those duplets occupy the same geometric position on the two cubes that are generated from each other by Dturn = ±1/3 and Dturn = ±2/3 **(Table 2, Figure 24, Figure 25)**.

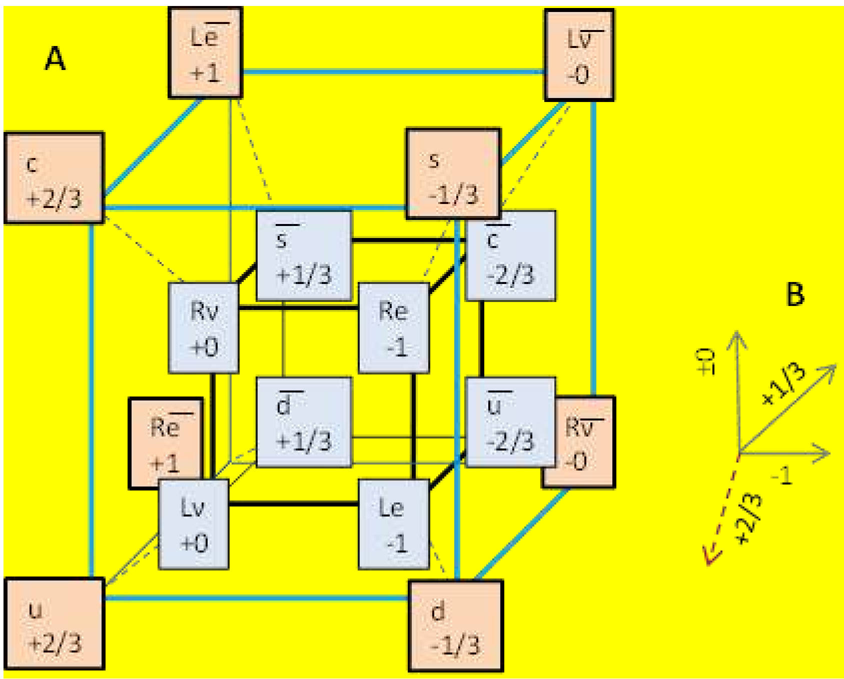


Figure 25. The fermion/fermion duplets depicted on the vertices of a 4D cube (A). The fourth-dimensional extension is represented by the inner/external placement of two 3D cubes.
Extension in the fourth-dimension is represented by dashed line of the coordinate system of the 4D q-cube **(B).**

The electric charge q of the duplets is the sum of two a-bits, q = a1 + a2, therefore the value added during the increase of dimensions could change the value of a1, a2, or both. Adding 0 can only cause a combined, opposite change in a1 and a2. During the increase of dimensions, the a1-bit changes, just as diametric rotation produces a lepton chart from the quark chart by adding ±1/3 or ±2/3 to the value of a1 **(Table 2, Figure 10)**. If we observe the effect of increasing dimensions on the change of a-bits **(Figure 26)**, it can be seen that the increase of dimensions and diametric rotation have the same result.

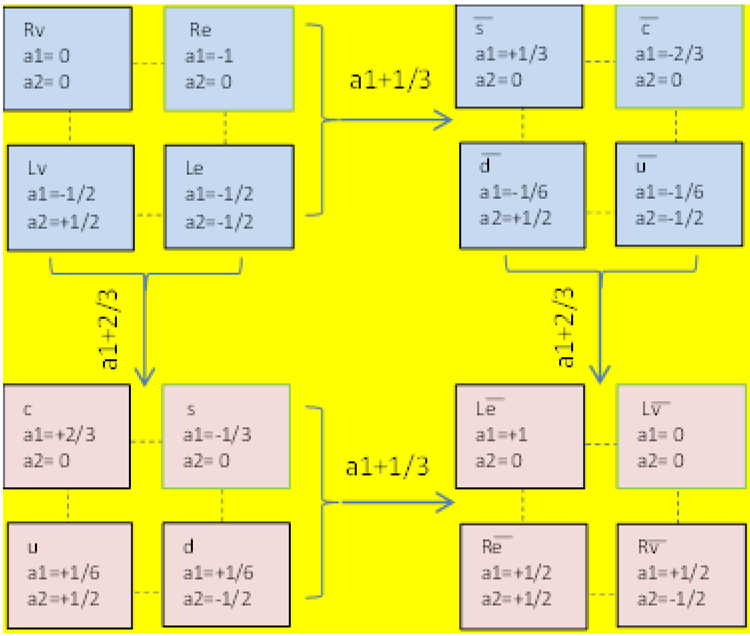


Figure 26. The changes of the a1 and a2 bits of the fermion/fermion duplets depicted on the four parallel faces of the 4D q-cube **(Figure 25)**, in the dimensions ±1/3 and ±2/3.

The 4D q-cube on the surface of a torus

The edge framework of the 4D cube corresponds to a torus coordinate system that contains four toroidal circular axes and four poloidal circular axes. This is illustrated by a stylized square torus **(Figure 27/A, B)**. As a result of the cube-torus transformation, the chirality of the blue cube reversed. Furthermore, for visual effect, some edges of the blue and black cubes have been bent, but these changes do not affect the connections of the vertices.

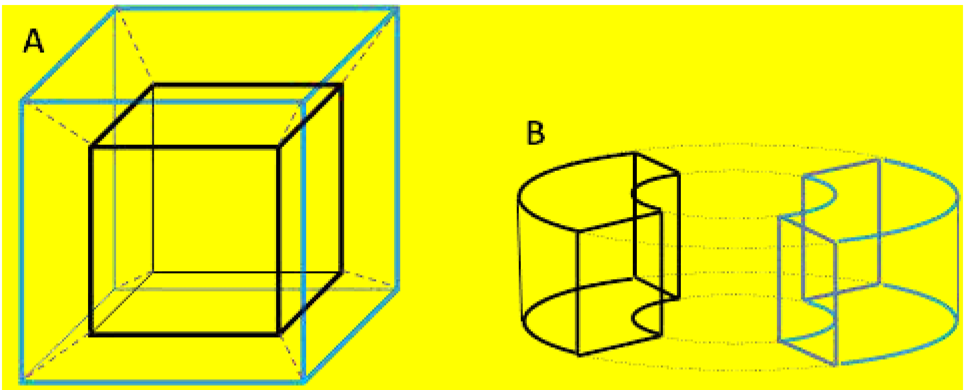


Figure 27. The edge frame of a 4D cube (A) and toroidal-poloidal coordinate system drawn on the surface of a square torus (B). The cube's two disjunct 3D subspaces (black and blue) correspond to similarly colored shapes on the torus. The cube's extent in the 4th dimension and the corresponding arcs on the torus are shown with dashed lines.

The quark, quark, lepton, lepton duplets projected onto the torus from the vertices of the 4D cube, are located on separate poloidal quadrangles, which are transformed into each other by toroidal rotations **(Figure 28)**. Starting from the poloidal square of the leptons, the poloidal squares of quarks, leptons, and quarks with q values follow each other on the torus in the toroidal direction with differences of -2/3, -1/3, +2/3, +1/3.

The charges of the duplets projected onto the torus from the vertices of the 4D cube are transformed by reflection, rotation, and translation operations into their pairs related by ide, alt, com, opp.

A translation parallel to the torus's axis of rotation designates the pairs in the ide relation. As a consequence of the ide relation (equal charges), the pairs in the alt, com, opp relations are also implied by two different geometric transformations.

Reflection about the center of the poloidal quadrilateral as well as translation perpendicular to the axis of rotation (radial direction) identifies the pairs related by the alt relation, while reflection about the axis of rotation and reflection about the center of the torus identifies the pairs related by the com relation. Translation perpendicular to the axis of rotation, equal to the diameter of the toroidal central circle, or double reflection performed about both the center of the torus and the center of the poloidal quadrilateral transforms the duplets related by the opp relation into each other. The translations must be performed in the appropriate direction.

The distinction between –0 and +0 is consistently implied by all four relations.

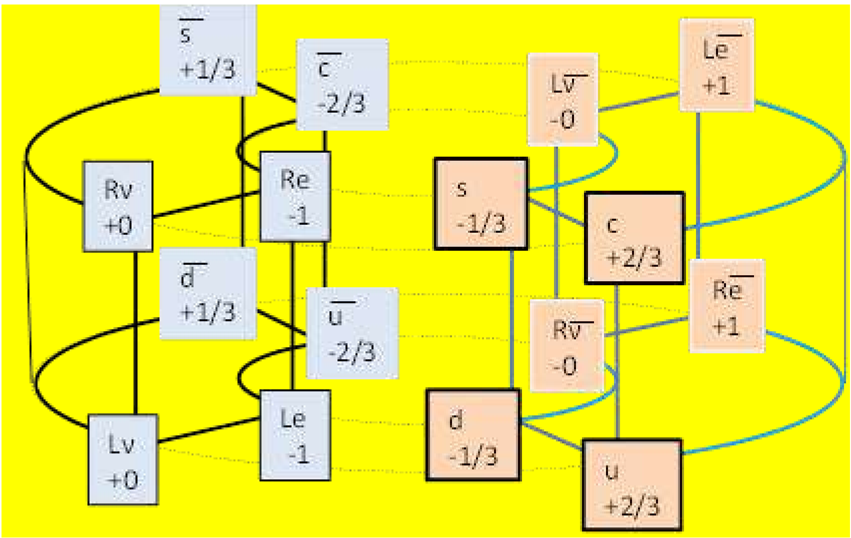


Figure 28. The location of the fermion/fermion duplets and value of their electric charge on the surface of a square torus.

On the torus, the C.I.A.O. relations between the charges of the duplets correspond to geometric transformations; therefore, the relations between ide, opp, alt, com also hold for these geometric transformations.

For example, according to the equality x* alt * com = x* opp, reflecting a charge value x first to the center of the poloidal circle and then to the center of the torus gives the opposite of x.
Hence, in this system, the C.I.A.O. relations and the geometric transformations can be used interchangeably.

Insertion of the charm/charm quark duplets and the lepton/lepton duplets into the SD graph

The charm quark step (Y' = +2/3, I = 0) is an alternative form of the strange quark step (Y' = –1/3, I = 0), so their starting and ending points are the same. Therefore, three new quark steps with negative rotation, proceeding together with the strange quark steps, represent them in every zone of the SD graph **(Figure 29)**. However, we assign them a positive rotation, i.e., a rotation value of Y' = +2/3.
When traversing the SD graph, we can use the strange quark step instead of the charm quark step if we add the number of charm quark steps to the rotation Y' of the graph path afterwards.
With the three new quark steps, we have formally expanded each zone of the SD graph with a fourth hyper edge. The fourth hyper edge connects the three nodes of the zone.

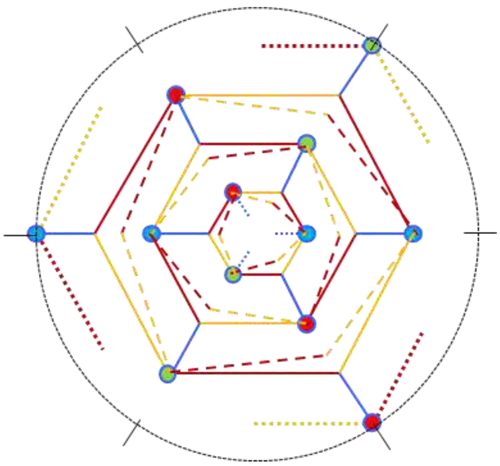


Figure 29. The SD-graph extended with charm quark steps. Charm quark steps are marked by dashed orange-red lines (--------).

In the quark set expanded by the charm quark step, the previously established relationships between stability and color charge remain valid for the quark step sequences corresponding to hadrons **(1)**. This can be seen from the fact that if we replace any number of strange quark steps in a SD meson graph path or in SD baryon graph paths with charm quark steps, the starting and ending point of the path remains unchanged. Each replacement causes a +1 change in the value (Y'), so the value q = Y' + I remains an integer.

If we assign the same color charge to strange quark steps and charm quark steps in the same direction between the same vertices, then the color neutrality of baryons still holds.

A lepton/$\overline{\text{lepton}}$ couple is defined by their bits on the SD graph. Starting from any node, the value a1 of the couple gives the angular rotation of the couple, while the value a2 gives its radial displacement. With the steps corresponding to any lepton/$\overline{\text{lepton}}$ couple, we reach the same diameter of the graph.

By plotting the quark/$\overline{\text{quark}}$ couples and lepton/$\overline{\text{lepton}}$ couples, we can model the β+ and β- decay of particles on the SD graph. We can verify that starting from any node, a single up quark step leads to the same place as the successive steps of a down quark + $\overline{\text{Re}}$ + Lν sequence. Likewise, a single down quark step leads to the same place as the successive steps of an up quark + Le + R$\overline{\nu}$ sequence.

Table of the elementary fermion duplets

Fermion couples identified with the vertices of four crosses (a1, a2) are located at lattice points within a lattice coordinate system in a square region (|a1| + |a2|≤ 1), where the numbers of horizontal and vertical lattice lines have the same parity **(Figure 30)**. Coordinate value of 5/6 that is excluded by point 2 of the diametric replacement lie outside the region.

In the figure, the vertices of the four crosses correspond to different colored couple types, and the fermions are [·]-symmetric for the corresponding fermions.

The duplets are located along the diagonal lines of the table at coordinate sums corresponding to their electric charge q = |a1| + |a2| = -1, -2/3, -1/3, 0, +1/3, +2/3, +1.

Different types of couples are arranged periodically on the vertical coordinate lines numbered 0, 1, 2,...,12. On lines numbered 0(mod3), 1(mod3), 2(mod3), we find lepton/$\overline{\text{lepton}}$, quark, $\overline{\text{quark}}$ duplets respectively.

Considering the pairs a1, a2 as vector coordinates, the sums of quark duplet + quark duplet and lepton- duplet + lepton duplet are always lepton/lepton duplets if the system is supplemented with the (0, -1) and (0, +1) reverse lepton duplets, as we have seen earlier **(Table 7)**.

We emphasize that only the results of these two additions always remain within the plane table. Addition between other kinds of couples can be defined only if the horizontal grid lines and vertical grid lines are circularized, turning the table into a torus surface. Thus, the addition of any two fermion or anti-fermion duplets results in a lattice point.

We can therefore conclude that quark/quark duplets and the lepton/ lepton duplets are those a1, a2 pairs which, even after opening the torus, form a closed algebraic structure with respect to addition defined on the torus.

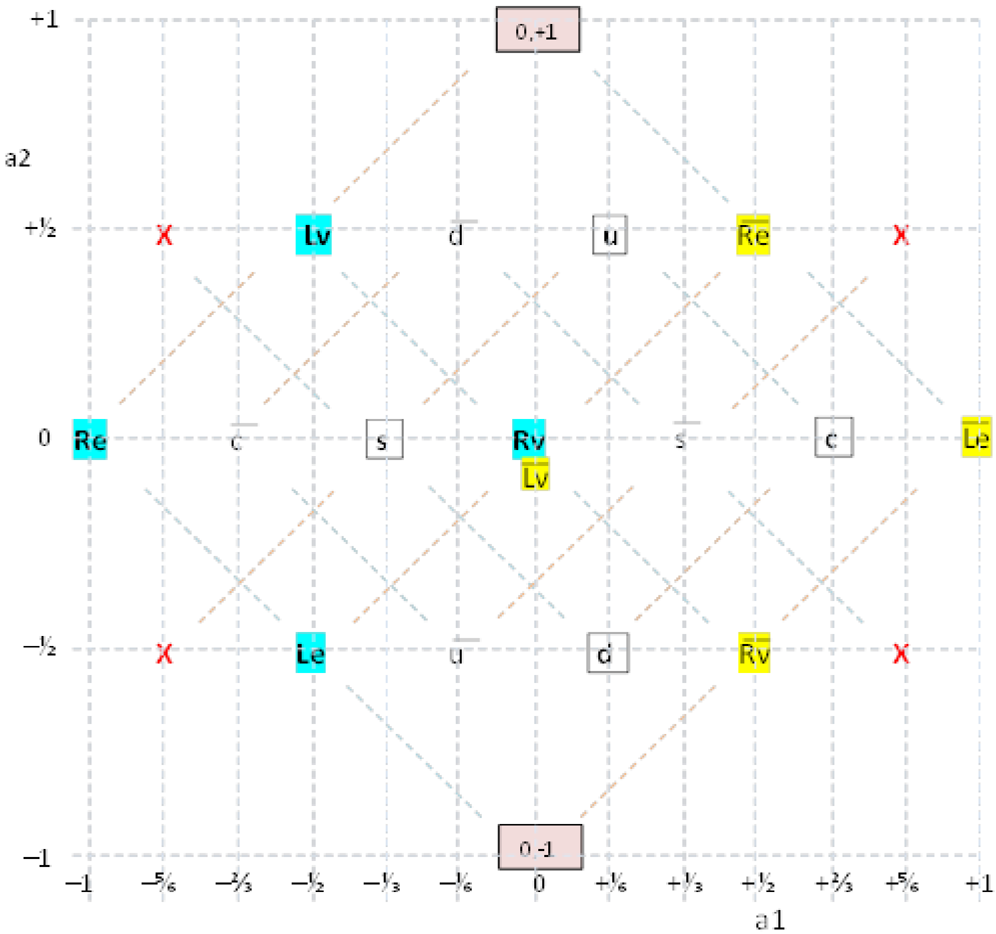


Figure 30. Table of elementary fermion duplets. The horizontal coordinates are the a1-bits, vertical coordinates are the a2-bits of the duplets. Quark duplets (framed), quark duplets (bare letters), lepton duplets (blue), lepton duplets (yellow) are placed to coordinates corresponding to their (a1, a2)-bits. Positions excluded by the point 2 of the diametric rules are red X-s. Positions (0, -1) and (0, +1) are red.

Duplets with equal q charges (-1, -2/3, -1/3, 0, +1/3, +2/3, +1) are connected by blue dashed diagonal lines. Lepton/lepton duplets, quark duplets, quark duplets fit to separated anti diagonal orange lines.

By closing the a1 and a2 axes into a circle, we projected the table **(Figure 30)** onto a torus, where the a1 coordinate axis represents the poloidal coordinate axis of the torus, the a2 axis represents the toroidal coordinate axis, and the bits of the duplets correspond to the coordinate values **(Figure 31)**. The sections of the torus are represented by the vertices of equilateral hexagons. On the sections, the a1 values vary in steps of 1/6 ($-1 \leq a1 \leq +1$) in the poloidal direction. The a2 values ($-1 \leq a2 \leq +1$) change in steps of 1/2 in the toroidal direction.

The fermion duplets may correspond to points on the outer surface of the torus **(Figure 31)**, or to points on both the outer and inner surfaces of the torus **(Figure 32)**. The two figures illustrate the sections of the torus containing the a2 values, which correspond to the rows of the fermion table.

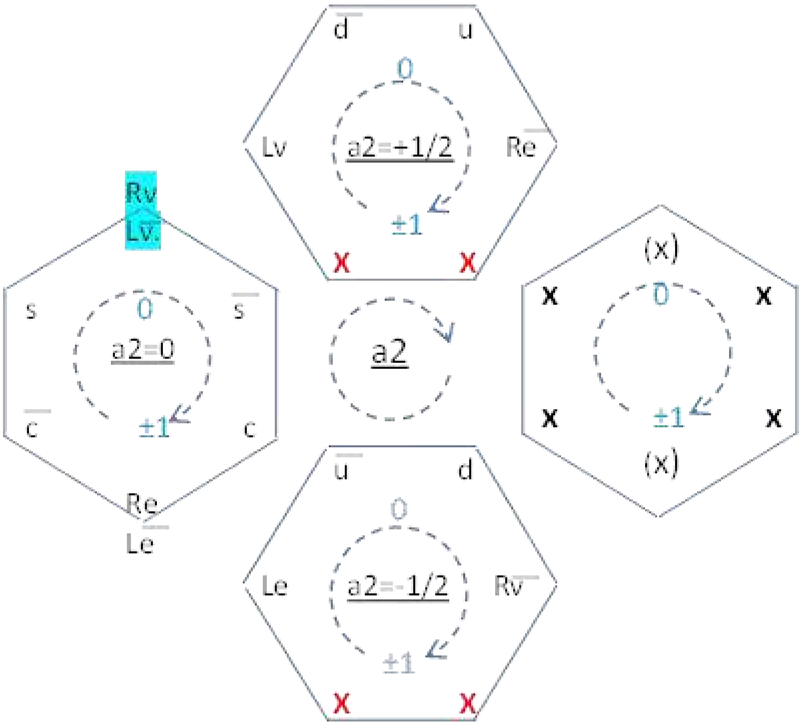


Figure 31. Elementary fermion duplets on the surface of a torus. The poloidal circle of the torus is used as the circular a1 axis, while the toroidal circle is used as the circular a2 axis. The origin (a1 = 0, a2 = 0) is highlighted in turquoise. The four hexagons represent the intersections of the torus at toroidal coordinates a2 = -1/2, 0, +1/2, ±1. At these four intersections, the fermion duplets are labeled with abbreviated names. Positions excluded according to point 2 of the diametric rules are marked with red Xs. Black Xs indicate the positions of missing duplets.

Fermion duplets with the same charge q = a1+a2 are indicated by diagonal lines on the fermion table. These lines can be found on the torus by moving from any fermion duplet +3 x 1/6 units in the poloidal direction, -1x 1/2 units in the toroidal direction, or multiples of both. In the figure, red X marks the coordinate positions where point 2 of the diametric replacement rule excludes the occurrence of a fermion duplet.

Equipotential lines (places with equal q) only remain unbroken on the surface of the torus if we allow a fourth toroidal coordinate value, a2 = +1 or a2 = -1, which corresponds to the fourth intersection of the torus **(Figure 31)**. The coordinate axis is a circle, therefore a2 = +1 and a2 = -1 coincide. For the intersection with coordinate a2 = ±1, we temporarily mark X's at positions a1 = ±1, ±2/3, ±1/3, 0.

By occupying the positions marked with X, the equipotential lines touch all four cross-sections, and after two revolutions (1 poloidal + 1 toroidal), they wind around the torus, returning to themselves.

In the torus cross-sections, after completing one circle, the poloidal a2 coordinate changes by 2 units, which corresponds to 2 circles, since we selected the circle as the unit. The contradiction is resolved if, instead of a circle, the duplets corresponding to the torus cross-section are positioned on the edge of a Möbius strip. In this case, the 2 units correspond to completing two circles.

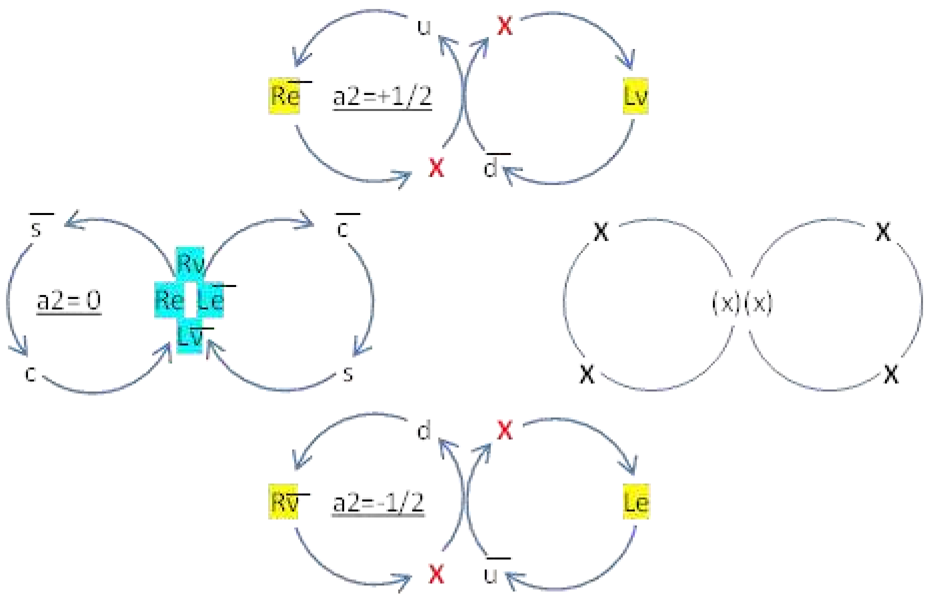


Figure 32. The location of the fermion/fermion duplets on the toroidal cross-sections of a torus with coordinates a2 = -1/2, 0, +1/2. The two members of each circle pair represent two poloidal circles running along the outer and inner surfaces of the torus. The contact point in the circle pairs is the center of the horn torus, the origin of the coordinate system. In these circle pairs, the right-hand circle has negative coordinate values (-1 to 0), while the left-hand circle has positive coordinate values (0 to +1). Blue directed 2π/3 arcs indicate the direction of the increase of a1. The a1 components of the fermion duplets, marked with the traditional abbreviation, correspond to the poloidal coordinates, and the a2 components correspond to the toroidal coordinates. L lepton duplets and R lepton duplets are turquoise; R lepton duplets and L lepton duplets are yellow. Positions excluded by Rule 2 of the diametric replacement are marked with red Xs. Black Xs indicate the positions of missing duplets.

For the sake of simpler representation, the edge of the Möbius strip is replaced by a figure-eight **(Figure 32)**. Consider the two linked circles as running along the inner and outer surfaces of the torus; thus, the a1 values vary in 1/3 increments ($-1 \leq a1 \leq +1$) in the poloidal direction, completing a loop on both the outer and inner surfaces of the torus.

The left-hand circles of the pairs **(Figure 32)** run along the outer surface of the torus, while the right-hand circles run along the inner surface. In this case, the origin can be taken as the center of a horn torus, and the fermion duplets are positioned on poloidal circle pairs with toroidal coordinates a2 = -1/2, 0, +1/2, ±1. The a1 coordinates of the point pairs on the inner and outer surfaces that touch each other are alternative values. They correspond to arcs drawn in the opposite direction to a point of a circle. The a2 coordinates of the point pair are equal.

The interpretation of the SD network

A stylized torus **(Figure 32)**, when cut along a poloidal circle, yields a cylinder. One surface of the cylinder corresponds to the inner surface of the torus, while the other surface corresponds to the outer surface. On both surfaces, we see the SD-graph stretched over the cylinder, which we have represented in a plane **(Figure 33)**.

The fermion/fermion duplets are located at the vertices of the two-layer SD-graph, which fit together on the two surfaces of the cylinder. The structure of the double SD-graph, the existence of vertices fitting its third diameter, necessitates the supplementation of the system of fermion/fermion duplets with lepton/$\overline{\text{lepton}}$ duplets, and prescribes their (y, i) coordinates, that is, the values of the a-bits of the lepton/lepton duplets.

Along the three diameters of the double SD-graph, lepton/anti-lepton duplets, quark duplets, and anti-quark duplets are arranged separately from each other.

The "wrong-handed" lepton/anti-lepton duplets gather at the (0,0) origin.

In the SD-graph, the different step-composition stable graph paths (returning to the same diagonal) correspond to hadron particles **(1)**. The quark steps that make up the graph paths are characterized by angular and radial quantum numbers of rotation (y) and distance (i).

Here we can also see that the quark steps correspond to the vertices of the double SD-graph that lie on its two diameters, whose (y) and (i) values are the a-bits of the quark/quark duplets (relative to the coordinates (a1 = 0, a2 = 0) of the 'wrong handed' lepton/lepton duplets).

The (y, i) quantum numbers (the a-bits of the duplets) interpreted on the SD-graph are this time coordinates within the torus and the SD-graph coordinate system. One layer of the SD-graph contains duplets with $a1 \leq 0$, while the other layer contains duplets with $0 \leq a1$.

The angular coordinate has two alternative values depending on the direction. In one layer, the plus direction is valid, while in the other layer, the minus direction is valid. Thus, the a1 coordinates of the point pairs of the two layers that fit together are alternating values.

The double SD graph eliminates the inconvenience experienced during the insertion of the c duplet and $\bar{c}$ duplet, where the endpoints of the quark steps coincided with those of the s and $\bar{s}$ duplets. In the double SD graph, the positions of c and s as well as $\bar{c}$ and $\bar{s}$ are identical, but they are located in different layers of the double graph.

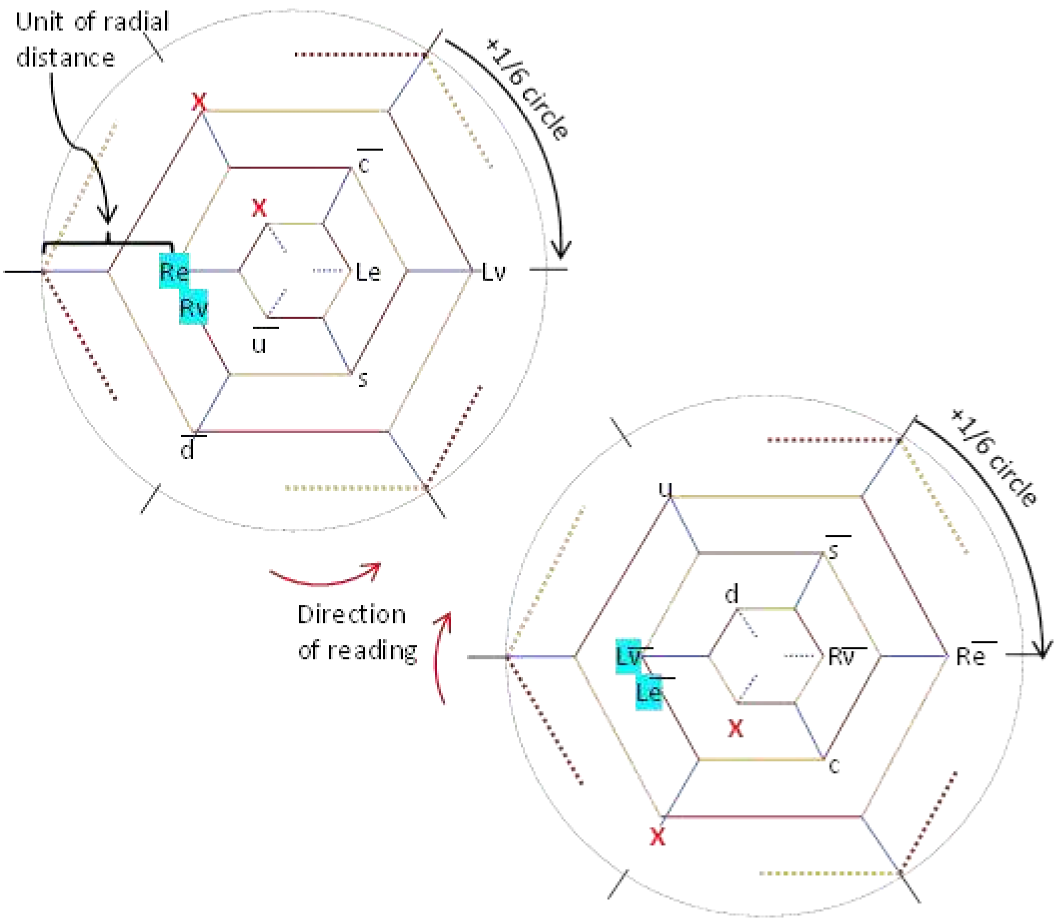


Figure 33. Fermion/fermion duplets on the vertices of the SD-graph. One SD-graph is located on the inner surface of a torus, while the other SD-graph is situated on the outer surface of a torus. The zones of the graphs correspond to the poloidal circles (with a2 = -1/2, 0, +1/2) shown in **Figure 32**, and to the rows of the fermion/fermion table **(Figure 30)**. The (a1, a2)-bits of the fermion duplets correspond to the SD-coordinates (y, i) relative to the origin (0,0). The positive direction is clockwise. The vertex (0,0), which represents the position of lepton duplets with "wrong handedness" (Re, Rν, Le, Lν), is highlighted in turquoise. The distance unit (i) is indicated by a bracelet. X symbols mark the positions excluded according to Rule 2 of the diametric replacement.

Chart breeding – chart interference

In addition to the symmetries of the summation of fermion and fermion duplets **(Table 7)**, it also highlights the order prevailing within the charts and the connections between the charts, that the charts can be algebraically "bred".

Similar to how the matrix of differences of quark a1-bits **(Figure 10/D, E)** created the lepton and lepton chart, we interpret the summation of the rows of two charts. The result of the operation is a matrix whose i, j element is the sum of the i-th element of one row and the j-th element of the other row.

Two charts are called mutually interferable charts if the value of Γ2 in their chart expression is equal but has opposite signs. The summation of the corresponding rows of two interferable charts as described above is called chart interference (chart ⌂ chart), which results in three identical interference matrices, that is, three identical interference charts.

As an example, we present the interference of two charts **(Figure 34, 35)**, where the two interferable charts are depicted as pages of a half-open book. If we sum the corresponding two a-bits at the intersections of the perpendiculars projected onto the a-bits, then in the space stretched between the opening sides of the book, three identical interference charts appear, like a hologram.

In the following, for easier representation, in chart expressions and charts we have displayed six times the values of the a-bits so that only whole numbers appear in the figures.

The chart ⌂ chart interference is an operation performed with the Γ coefficients in chart expressions, the result of which can also be expressed with the Γ coefficients.

In the operation "chart1 ⌂ chart2 = interference charts3," the chart1 Γ1 coefficient is the chart3 Γ1 coefficient, and the negation of the chart2 Γ1 coefficient is the chart3 Γ2 coefficient. The result of the interference operation (charts3) therefore depends only on Γ1 coefficient of chart1 and chart2.

In the left-side and right-side interference charts, Γ1 and Γ2 are swapped, and their signs change to the opposite. The operation chart1 ⌂ chart2 = chart3 is commutative if and only if the two Γ1 values are equal; therefore, it is essential to preserve the orientation of the elements. A rectangular coordinate system shows the position and orientation of chart1, chart2, chart3 **(Figure 34/B)**. The interference of two charts always produces three charts, as indicated by the plural form.

In the quark chart ⌂ quark chart = lepton charts **(Figure 34/A)**, lepton chart ⌂ lepton chart = lepton charts **(Figure 35/A)** interference diagrams, the a-bits of the charts are also indicated along with the chart expressions (6x values).

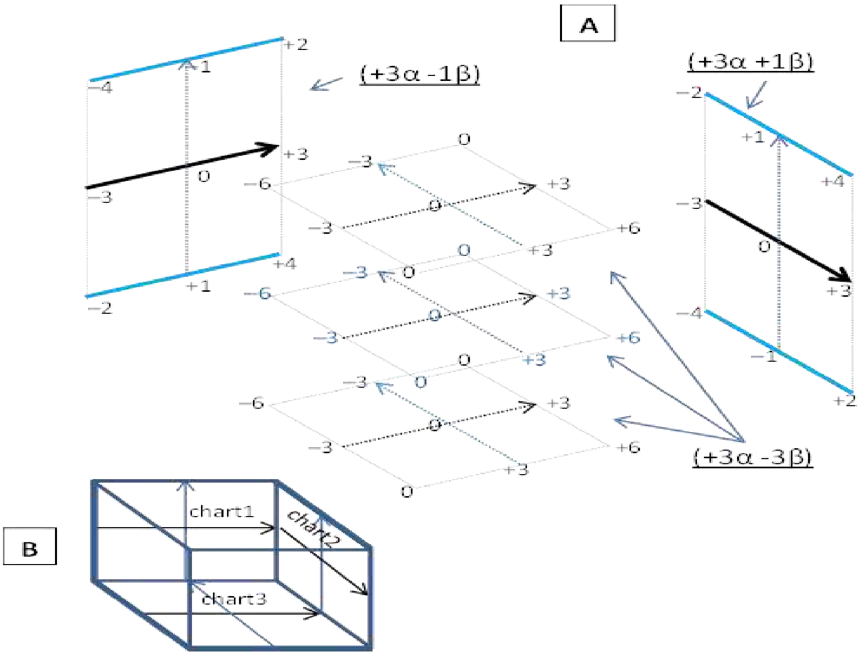


Figure 34. Chart ⌂ chart interference (3α -β) ⌂ (3α +β) = (3α -3β) (A).
The terms of the operation chart1 ⌂ chart2 = chart3 are placed on the inner surface of a cuboid (B). Back face: chart1, right face: chart2, bottom face: chart3 (interference charts). The α axis of the charts is black, the β axis is blue.

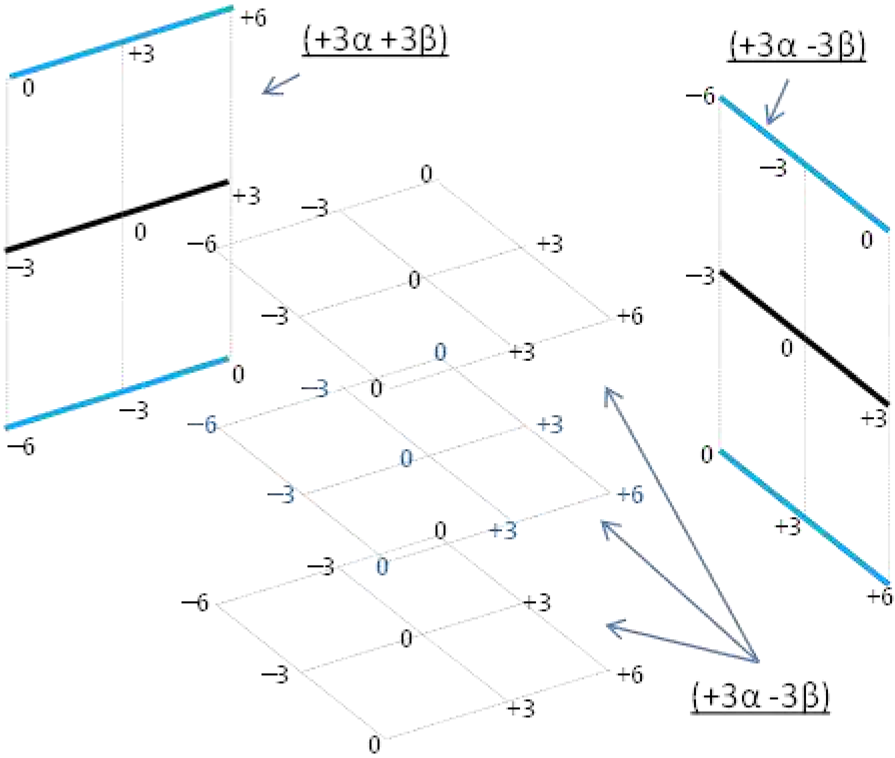


Figure 35. Chart ⌂ chart interference (3α +3β) ⌂ (3α -3β) = (3α -3β).

Using the [ı]-chart notation for the mirror image of a chart, the quark chart and the [ı]-quark chart are interferable with the quark chart and the [ı]- quark chart. The lepton chart and the [ı]-lepton chart are interferable with the lepton chart and the [ı]-lepton chart **(Table 11)**.

The interferable and non-interferable relationships are summarized in **Figure 36**. The interference chart is always a lepton/lepton chart or their [ı]-mirror image.

A Chart interferences (±3α -1β) ⌂ (±3α +1β) from both sides

chart1		chart2		interference	chart2		chart1		interference
+3α−β −4 −1 +2 −3 0 +3 −2 +1 +4	⌂	+3α+β −2 +1 +4 −3 0 +3 −4 −1 +2	=	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6	+3α+β −2 +1 +4 −3 0 +3 −4 −1 +2	⌂	+3α−β −4 −1 +2 −3 0 +3 −2 +1 +4	=	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6
+3α−β −4 −1 +2 −3 0 +3 −2 +1 +4	⌂	−3α+β +4 +1 −2 +3 0 −3 +2 −1 −4	=	+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0	−3α+β +4 +1 −2 +3 0 −3 +2 −1 −4	⌂	+3α−β −4 −1 +2 −3 0 +3 −2 +1 +4	=	−3α−3β 0 −3 −6 +3 0 −3 +6 +3 0
−3α−β +2 −1 −4 +3 0 −3 +4 +1 −2	⌂	+3α+β −2 +1 +4 −3 0 +3 −4 −1 +2	=	−3α−3β 0 −3 −6 +3 0 −3 +6 +3 0	+3α+β −2 +1 +4 −3 0 +3 −4 −1 +2	⌂	−3α−β +2 −1 −4 +3 0 −3 +4 +1 −2	=	+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0
−3α−β +2 −1 −4 +3 0 −3 +4 +1 −2	⌂	−3α+β +4 +1 −2 +3 0 −3 +2 −1 −4	=	−3α+3β +6 +3 0 +3 0 −3 0 −3 −6	−3α+β +4 +1 −2 +3 0 −3 +2 −1 −4	⌂	−3α−β +2 −1 −4 +3 0 −3 +4 +1 −2	=	−3α+3β +6 +3 0 +3 0 −3 0 −3 −6

B Chart interferences (±3α +3β) ⌂ (±3α -3β) from both sides

chart1		chart2		interference	chart2		chart1		interference
+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0	⌂	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6	=	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6	⌂	+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0	=	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6
+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0	⌂	−3α−3β 0 −3 −6 +3 0 −3 +6 +3 0	=	+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0	−3α−3β 0 −3 -6 +3 0 -3 +6 +3 0	⌂	+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0	=	−3α−3β 0 −3 −6 +3 0 −3 +6 +3 0
−3α+3β +6 +3 0 +3 0 −3 0 −3 −6	⌂	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6	=	−3α−3β 0 −3 −6 +3 0 −3 +6 +3 0	+3α−3β −6 −3 0 −3 0 +3 0 +3 +6	⌂	−3α+3β +6 +3 0 +3 0 −3 0 −3 −6	=	+3α+3β 0 +3 +6 −3 0 +3 −6 −3 0
−3α+3β +6 +3 0 +3 0 −3 0 −3 −6	⌂	−3α−3β 0 −3 −6 +3 0 −3 +6 +3 0	=	−3α+3β +6 +3 0 +3 0 −3 0 −3 −6	−3α−3β 0 −3 −6 +3 0 −3 +6 +3 0	⌂	−3α+3β +6 +3 0 +3 0 −3 0 −3 −6	=	−3α+3β +6 +3 0 +3 0 −3 0 −3 −6

Table 11. The (±3α -3β) ⌂ (±3α +3β) chart interference (A) and the (±3α +3β) ⌂ (±3α -3β) chart interference (B). The operations performed in the reverse order are shown in the second half of the two figures. The first row of the charts is the chart expression. The meaning of the colors: Γ2 = -1 (blue), Γ2 = +1 (green), Γ2 = -3 (white), Γ2 = +3 (yellow).

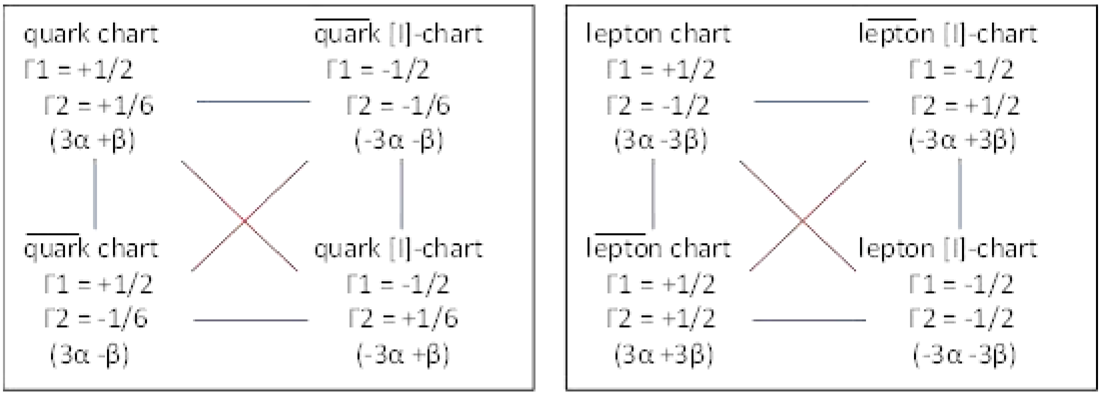


Figure 36. The interference of the quark chart, the quark chart [–]-mirror image (quark chart), the quark chart [ı]-mirror image, and the quark chart [–]*[ı]-mirror image, as well as the lepton chart, the lepton chart [–]-mirror image (lepton chart), the lepton chart [ı]-mirror image, and the lepton chart [–]*[ı]-mirror image. The charts that can interfere are connected with a blue line, and the charts that cannot interfere are connected with a red line. The [ı]-chart means the [ı]-mirror image chart.

The interference of quark/$\overline{\text{quark}}$ charts and lepton/$\overline{\text{lepton}}$ charts, due to their $\Gamma 1 = \pm 1/2$ values, always produces lepton/$\overline{\text{lepton}}$ charts ($\Gamma 2 = \pm 1/2$). To balance the demographic disparity, we expanded the chart population with charts having $\Gamma 1 = \pm 1/6$ so that charts with $\Gamma 2 = \pm 1/6$ would not be eliminated during chart breeding. The rules of interference remain unchanged after this expansion. The interferencies of the expanded chart set are contained in the operation tables **(Table 12)**.

A

⌂	−1α −1β	+1α −1β	−3α −1β	+3α −1β
−1α +1β	−1α +1β	−1α −1β	−1α +3β	−1α −3β
+1α +1β	+1α +1β	+1α −1β	+1α +3β	+1α −3β
−3α +1β	−3α +1β	−3α −1β	−3α +3β	−3α −3β
+3α +1β	+3α +1β	+3α −1β	+3α +3β	+3α −3β

B

⌂	−1α +1β	+1α +1β	−3α +1β	+3α +1β
−1α −1β	−1α +1β	−1α −1β	−1α +3β	−1α −3β
+1α −1β	+1α +1β	+1α −1β	+1α +3β	+1α −3β
−3α −1β	−3α +1β	−3α −1β	−3α +3β	−3α −3β
+3α −1β	+3α +1β	+3α −1β	+3α +3β	+3α −3β

C

⌂	−1α −3β	+1α −3β	−3α −3β	+3α −3β
−1α +3β	−1α +1β	−1α −1β	−1α +3β	−1α −3β
+1α +3β	+1α +1β	+1α −1β	+1α +3β	+1α −3β
−3α +3β	−3α +1β	−3α −1β	−3α +3β	−3α −3β
+3α +3β	+3α +1β	+3α −1β	+3α +3β	+3α −3β

D

⌂	−1α +3β	+1α +3β	−3α +3β	+3α +3β
−1α −3β	−1α +1β	−1α −1β	−1α +3β	−1α −3β
+1α −3β	+1α +1β	+1α −1β	+1α +3β	+1α −3β
−3α −3β	−3α +1β	−3α −1β	−3α +3β	−3α −3β
+3α −3β	+3α +1β	+3α −1β	+3α +3β	+3α −3β

Table 12. The operation tables of the ⌂ interference of $\Gamma 1$ = +1, -1, +3, -3 value interferable charts. The headers of the rows and columns show the two interfering charts, and in the row-column intersections, the chart expression of their interference result is shown. The result of the interference operation is three identical charts, so every element in the tables represents three identical chart expressions. The ⌂ operation is not commutative; the tables next to each other (A and B, and C and D) show the results of the operation performed from two sides. The $\Gamma 2$ value of the charts is indicated by colors: $\Gamma 2$ = +1 (green), $\Gamma 2$ = -1 (blue), $\Gamma 2$ = +3 (yellow), $\Gamma 2$ = -3 (white).

As an example, we illustrated the (3α – 3β) ⌂ (-1α + 3β) = (3α + 1β) interference in the extended set of interferable charts.

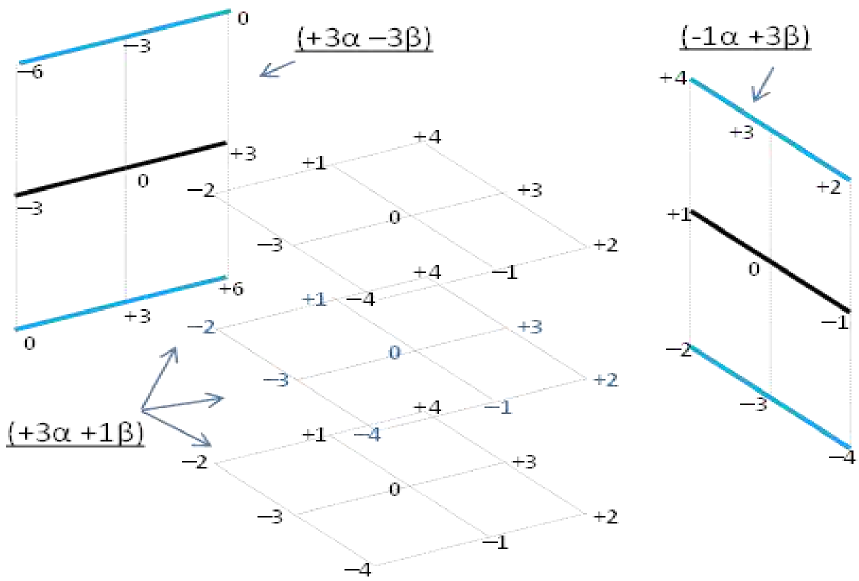


Figure 37. Chart ⌂ chart interference (3α -3β) ⌂ (-1α +3β) = (3α +1β).

Partition matrices

The interference of two charts consists of adding the a-bits of the two charts. Viewed in the opposite direction, these additions are two-member partitions of a-bits of three identical charts, which we compiled in tables, and next to the tables, we represented them on the partition matrix of the interference **(Figure 34, 35, 37, Table 13, 14, 15)**.

The creation of the partition matrix: the rows and columns of the matrix were labeled with integers, and in the initial state, every matrix element i, j was 0.

We increased the (i, j) element of the matrix by 1 whenever we added the a-bit of value i from chart1 and the a-bit of value j from chart2 in the interference operation. The matrix elements that were 0 were left empty in the figures.

The partition matrix of chart⌂chart interference can be decomposed into components consisting of three matrix elements, from which the full matrices can be algorithmically constructed. The (0,0) matrix element in the partition matrices is a nonzero element, and by generating 1 → 3 → 9 → 27 (not necessarily distinct) elements from it, we can produce the other nonzero elements of the matrices as well.

As a first step, we generated the four components of the chart1 ⌂ chart2 = charts3 interference partition algorithm. The first component is (0,0), which is the coordinate of the initial matrix element. The other components are formed from the coefficients chart1 $\Gamma1_1$ and $\Gamma2_1$ as well as chart2 $\Gamma1_2$ and $\Gamma2_2$, where the subscript indicates which chart the Γ coefficient belongs to in the operation. The components of the algorithm consist of the following terms enclosed in braces:

(0,0) **[**$(-\Gamma1_1,0)$, $(+\Gamma1_1,0)$**]** **[**$(0,+\Gamma1_2)$, $(0,-\Gamma1_2)$**]** **[**$(+\Gamma2_1,+\Gamma2_2)$, $(+\Gamma2_2,+\Gamma2_1)$**]**

During the execution of the algorithm, we add the next component's pair of values to the coordinates of the matrix element (0,0) one by one according to the rules of vector coordinate addition. This yields three matrix elements, whose values we increase by 1.

We then add the next component's pair of values to the coordinates of all three elements individually, which designates six elements, whose values we also increase by 1.

Finally, we add the next component's pair of values to the coordinates of every element affected so far, and increase the values of the elements designated in this way by 1.

At this point, the algorithm has been completed, and we have increased the values of the matrix elements by a total of 27. Every coordinate pair designated in the process represents the values of the two terms resulting from partitioning one of the values of the three identical interference charts.

The result of the procedure is independent of the order of the two terms within the parentheses, as well as the order of the three parenthetical components.

With this notation, the algorithmic generation of the (3α – β) ⌂(3α + β) = (3α - 3β) interference **(Figure 34)** partition matrix **(Table 13)**:

(0,0) **[**(-3, 0), (+3, 0)**]** **[**(-1,+1), (+1, -1)**]** **[**(0,+3), (0,-3)**]**,

the algorithmic generation of the (3α +3β) ⌂ (3α -3β) = (3α -3β) interference **(Figure 35)** partition matrix **(Table 14)**:

(0,0) **[**(-3,0), (+3,0)**]** **[**(0,+3), (0, -3)**]** **[**(-3,+3), (+3, -3)**]**,

the algorithmic generation of the (3α -3β) ⌂ (-1α +3β) = (3α +1β) interference **(Figure 37)** partition matrix **(Table 15)**:

(0,0) **[**(-3,0), (+3,0)**]** **[**(0,+1), (0,-1)**]** **[**(-3,+3), (+3, -3)**]**.

+6 =	**0 =**	**−6 =**
+4 +2	+4 −4	−2 −4
+3 +3	+3 −3	−3 −3
+2 +4	+2 −2	−4 −2
+3 =		**−3 =**
+4 −1	+1 −1	+1 −4
+3 +0	0 −0	0 −3
+2 +1	−1 +1	−1 −2
+1 +2	−2 +2	−2 −1
0 +3	−3 +3	−3 −0
−1 +4	−4 +4	−4 +1

Algorithm: (0,0) [(-3,0), (+3,0)] [(-1,+1), (+1, -1)] [(0,+3), (0,-3)]

Table 13. Decomposition of the interference charts (3α – 3β) produced by (3α – β) ⌂(3α + β) = (3α - 3β) interference **(Figure 34)**. Partition matrix, partition of a-bits and the algorithmic expression of the partition matrix. A detailed description of them can be found in the text.

+6 =	**0 =**	**−6 =**
0 +6	−6 +6	−6 −0
+3 +3	+3 −3	−3 −3
+6 +0	0 −0	0 −6
+3 =		**−3 =**
−3 +6	−3 +3	−6 +3
0 +3	0 −0	−3 −0
+3 −0	+3 −3	0 −3
0 +3	0 −0	−3 −0
+3 −0	+3 −3	0 −3
−3 +6	+6 −6	+3 −6

Algorithm: (0,0) [(-3,0), (+3,0)] [(0,+3), (0, -3)] [(-3,+3), (+3, -3)]

Table 14. Decomposition of the interference charts (3α – 3β) produced by (3α +3β) ⌂ (3α -3β) = (3α -3β) interference **(Figure 35)**. Partition matrix, partition of a-bits and the algorithmic expression of the partition matrix. A detailed description of them can be found in the text.

−4 =	**−3 =**	**−2 =**
−4 −0	−3 −0	−2 −0
−1 −3	0 −3	+1 −3
+2 −6	+3 −6	+4 −6
−1 =	**0 =**	**+1 =**
−4 +3	−3 +3	−2 +3
−1 −0	0 −0	+1 −0
+2 −3	+3 −3	+4 −3
+2 =	**+3 =**	**+4 =**
−4 +6	−3 +6	−2 +6
−1 +3	0 +3	+1 +3
+2 −0	+3 +0	+4 +0

Algorithm: (0,0) [(-3,0), (+3,0)] [(0,+1), (0,-1)] [(-3,+3), (+3,-3)]

Table 15. Decomposition of the interference charts (3α +1β) produced by (3α -3β) ⌂ (-1α +3β) = (3α +1β) interference **(Figure 37)**. Partition matrix, partition of a-bits and the algorithmic expression of the partition matrix. A detailed description of them can be found in the text.

Weak quantum numbers of the fermion/fermion duplets

So far, we have not clarified the inconsistency, that the diametric replacement of a0 = -1/3 produces a-bits corresponding to the hyper charge and isospin (Y', I) of quarks/quarks, while the diametric replacement of a0 = 0 produces a-bits corresponding to the weak hyper charge and weak isospin (Y'w, Iw) of leptons/leptons **(Table 17)**.

The apparent contradiction is resolved by the fact that the four quarks' (Y', I) values simultaneously correspond to the weak charges of the left- and right-handed down-type and up-type quarks (Y'w, Iw) as well **(Table 17)**. The table shows that the a1 and a2 bit pairs attributed to a single entity actually have three different carriers. Thus, the number of duplets under scope has tripled.

The a-bits of the d-quark duplet are carried by three down-type, left-handed quark duplets,

the a-bits of the s-quark duplet are carried by three down-type, right-handed quark duplets,

the a-bits of the u-quark duplet are carried by three up-type, left-handed quark duplets,

the a-bits of the c-quark duplet are carried by three up-type, right-handed quark duplets **(Table 17)**.

Each L and R lepton duplet shares its a-bits among three entities as well:

the a-bits of the Le duplet are shared by three duplets (Le, Lμ, Lτ),

the a-bits of the Re duplet are shared by three duplets (Re, Rμ, Rτ),

the a-bits of the Lν duplet are shared by three duplets ($L\nu_e$, $L\nu_\mu$, $L\nu_\tau$),

the a-bits of the Rν duplet are shared by three duplets ($R\nu_e$, $R\nu_\mu$, $L\nu_\tau$) **(Table 17)**.

For the anti-duplets, the opposite a1, a2 values and the opposite L/R handedness are assigned.

This resolves the apparent inconsistency that the quantum numbers (Y', I) and (Y'w, Iw), which participate in the strong and weak interactions respectively, appear together in the result of the diametric replacement. The a-bits created by the diametric replacement actually correspond to the Y'w, Iw charges. The charges of the strong and weak interactions are not mixed by the diametric procedure; rather, this is a property of the system being modeled.

Strong interaction			Fermion	Weak interaction				
				R		L		
Y'	I	Q_E		Y'_W	I_W	Y'_W	I_W	Q_W
+1/6 -1/3 (-1/3	-1/2 0 0)	-1/3	d s (b)	-1/3	0	+1/6	-1/2	-2/3
+1/6 +2/3 (+2/3	+1/2 0 0)	+2/3	u c (t)	+2/3	0	+1/6	+1/2	+1/3
0		-1	e μ τ	-1	0	-1/2	-1/2	-0
0		+0	νe νμ ντ	0	0	-1/2	+1/2	+1

Table 16. The quantum numbers (Y', I) and (Y'_W, I_W) of the fermions/fermions.

Coordinates of diametrically opposite points of a circle

In the diametric replacement of arc a0, in addition to the starting and ending points of the original arc, their diametrically opposite points on the circle play a similar role. To interpret their role, the relationship between the coordinates of a point on the circle and its diametric point must be examined.

Due to the two traversal directions of the circle, every point on the circle serves as the endpoint of two opposite-direction alternative arcs, therefore it has two different coordinates, x and (x + c) (where c = fc(x) = ±1 and -1 ≤ x ≤ 1), which are thereafter denoted by [x, (x + c)]. The signs of c and x are opposite.

The values of c = fc(x) are indicated in blue **(Figure 38)**. At the points x = -1, 0, 1, the function fc(x) has not been interpreted yet.

The diametrically opposite point of the circle point [x, (x + fc(x))] is [x + (d1 = fd(x)), (x + fc(x)) + (d2 = fd(x + fc(x)))]. The possible values of d are ±1/2.

The values d = fd(x) are marked in red **(Figure 38)**. At the positions x = -1, -1/2, 0, 1/2, 1, the function fd(x) has not yet been defined.

The property of fc(x) and fd(x) is that (x + fd(x)) and ((x + fc(x)) + fd(x + fc(x))) are alternative values, thus the sum of (x + fd(x)) and the corresponding c value is equal to ((x + fc(x)) + fd(x + fc(x))), so (x + fd(x)) + fc(x + fd(x)) = (x + fc(x)) + fd(x + fc(x))

The two expressions are symmetric with respect to the interchange of the functions fc and fd, making fc and fd dual of each other.

In the diametric replacement of a0 = -1/3 **(Figure 4)**, the endpoint of arc a1 is the point with coordinates [-1/3, +2/3] or the diametrically opposite [-5/6, +1/6], while the endpoint of a2 is [-1/3, +2/3]. The possibility a1 = -5/6 is excluded by point 2 of the rule for diametric replacement.

In the diametric replacements of the arc a0 = 0, the endpoint of arc a1 is the point [0, 0] or the diametrically opposite [-1/2, +1/2], while the endpoint of a2 is [0, 0].

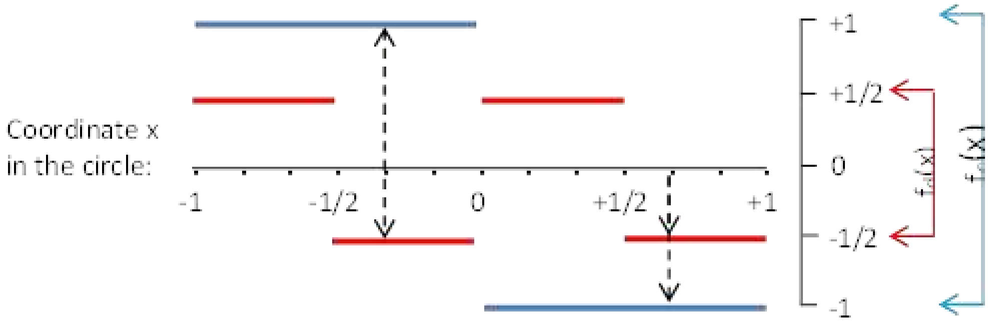


Figure 38. The function c = fc(x) (blue) describes the difference c between the positive and negative direction coordinates of point x (horizontal axis) on a circle. The function d = fd(x) (red) describes the difference d between the coordinate x of a point and the coordinate of the diametrically opposite point on a circle. Dashed arrows indicate the c and d values corresponding to the coordinates x = -1/3 and x = +2/3.

The endpoints a1 and a2 corresponding to the diametric replacements of a0 = 0 and a0 = -1/3 ([-5/6, +1/6], [-1/3, +2/3] and [-1/2, +1/2], [0, 0]) were plotted separately for the + and – coordinates on a circle with a + and a – orientation **(Figure 39/A, B)**. In other words, on one circle, the x-coordinate of the points [x, (x + c)] is shown, and on the other circle, the (x + c) coordinate is displayed.
The coordinates of diametrically opposite points are identically colored on the two circles.
In the figure, the coordinates of points at identical positions on the – and + oriented circles are alternative values. The coordinates of points in positions symmetric with respect to the 0__1/2 diameter are complementary values on both circles. The coordinates symmetric to each other with respect to the central axis parallel to the 0__1/2 diameter have opposite values.

The shape of the quantum number universe

If a double loop is formed from the line of the circle, it transforms into the edge of a Möbius strip **(Figure 39/C, D)**, where points diametrically opposite on the circle come next to each other on the same spoke of the Möbius strip.

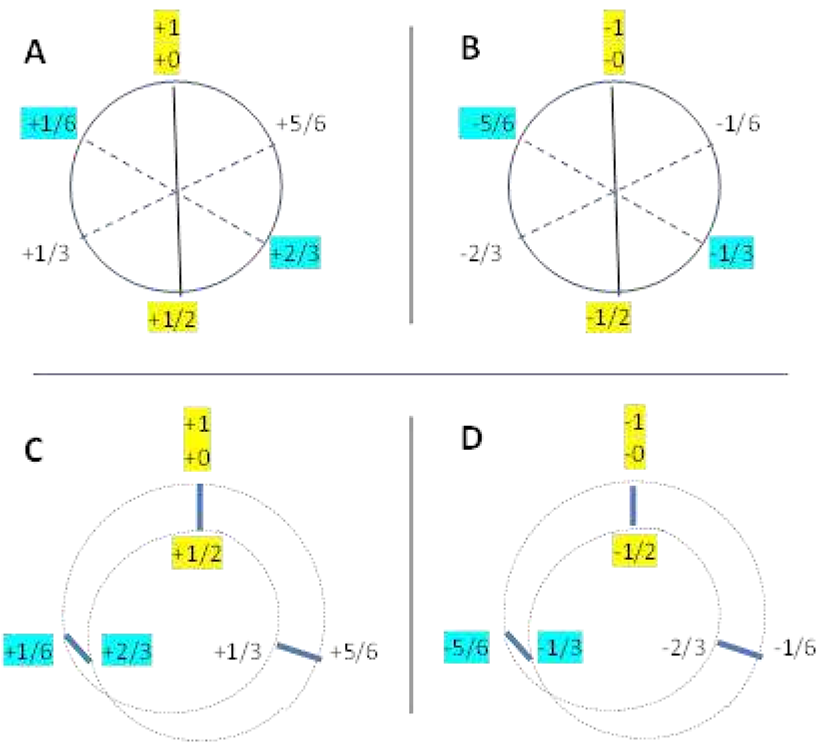


Figure 39. The coordinates of point x and its diametrically opposite point in the + and – traversal directions on two separate circles (A, B) and on the edges of two Möbius strips (C, D). x = 0 (turquoise), x = -1/3 (yellow), x = +1/3 (uncolored).

The starting point 0 and the ±1/3 endpoints of the prequark and its anti-quark divide a circle into three thirds. The double loop formed from this circle is the edge of a Möbius strip, on which the diametrically opposite points of the 0 and ±1/3 points are placed in distinguished positions, and these constitute the second endpoints of the spokes.
The 0 and ±1/3 points and their diametric points divide the original circle into six parts. We observe the same thing when opening the mini SD-graph, resulting in steps with 1/6 circle rotation **(Figure 1)**.

The surface enveloping the Möbius strip is a torus, and it is considered that the edge of the Möbius strip runs along the surface of the torus. The arcs running into two different endpoints of a spoke in opposite directions are called diametric alternative arcs, thereby distinguishing them from the circular alternative (alt) arc pairs. The difference of the circular alternative arcs is fc(x), i.e., ±1. The difference of the diametric alternative arcs is fd(x), i.e., ±1/2.

The functions fc(x) and fd(x) are not defined at integer and half-integer points, therefore the circular and diametric alternative arcs for a0 = 0 are generated using the same strategy applied in the diametric replacement of a0 = 0. That is, by adding +1/3 to the coordinates corresponding to the x = -1/3 spoke, we produce the coordinates corresponding to the x = 0 spoke. Consequently, we obtain the values x = 0, (x + fc(x)) = +1, (x + fd(x)) = -1/2, ((x + fc(x)) + fd(x + fc(x))) = +1/2, which are the coordinates of the endpoints of the arcs converging to the spoke 0 **(Figure 39/C, D)**.

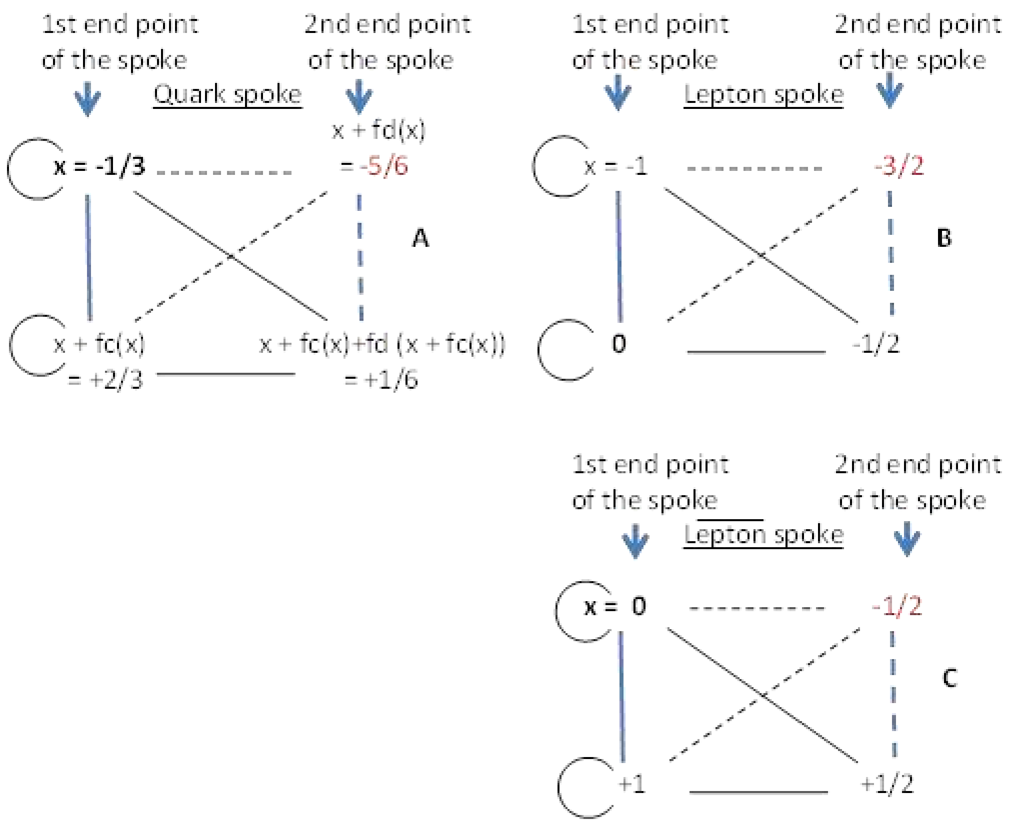


Figure 40. The two-directional coordinates of the 1st and 2nd endpoints of the quark spoke (A), the derived (D_{turn} = -2/3) lepton spoke (B), and the derived (D_{turn} = +1/3) lepton spoke (C) represent the $Y'w_L$ and $Y'w_R$ quantum numbers. The coordinate of a 1st endpoint is a $Y'w_R$ quantum number, and the coordinate of a 2nd endpoint is a $Y'w_L$ quantum number.
The arcs running from the spoke endpoints to the first endpoint (represented by continuous black lines) of the same spoke have magnitudes equal to the Iw_L and Iw_R quantum numbers. Black loops symbolize a 0 arc running from the point to itself.
The excluded a1 = 5/6 (red) arc and the arcs derived from it are indicated by dashed lines.

We represented the arcs a0 = -1/3 and a0 = +2/3 corresponding to the prequark and the alternative prequark **(Figure 41/A)**, as well as their diametric replacements (quark duplets) on the edge of a Möbius strip **(Figure 41/B, C)**. The diametric replacement adapted to the Möbius strip produces the quantum numbers Y', I (a1-, a2-bits) of the quark duplets and the quantum numbers Y'w, Iw of the fermion/fermion duplets.
Each a1-bit is an arc running from the 0 point to one of the quark spokes. The arcs running to the second end of the spokes arrive at the prequark's endpoint via a poloidal semicircle. These additional ±1/2 arcs are the a2-bits of the duplets.

The a2 bits correspond to the -1/2, 0, +1/2 arcs running along the poloidal circle of the Möbius strip/torus, distinguished in the planar diagrams by the line thickness.
Note that the a0 = +2/3 arc has only two regular diametric replacements **(Figure 41/C)**, which implies two additional quark duplets. Their analysis will be addressed on another occasion.

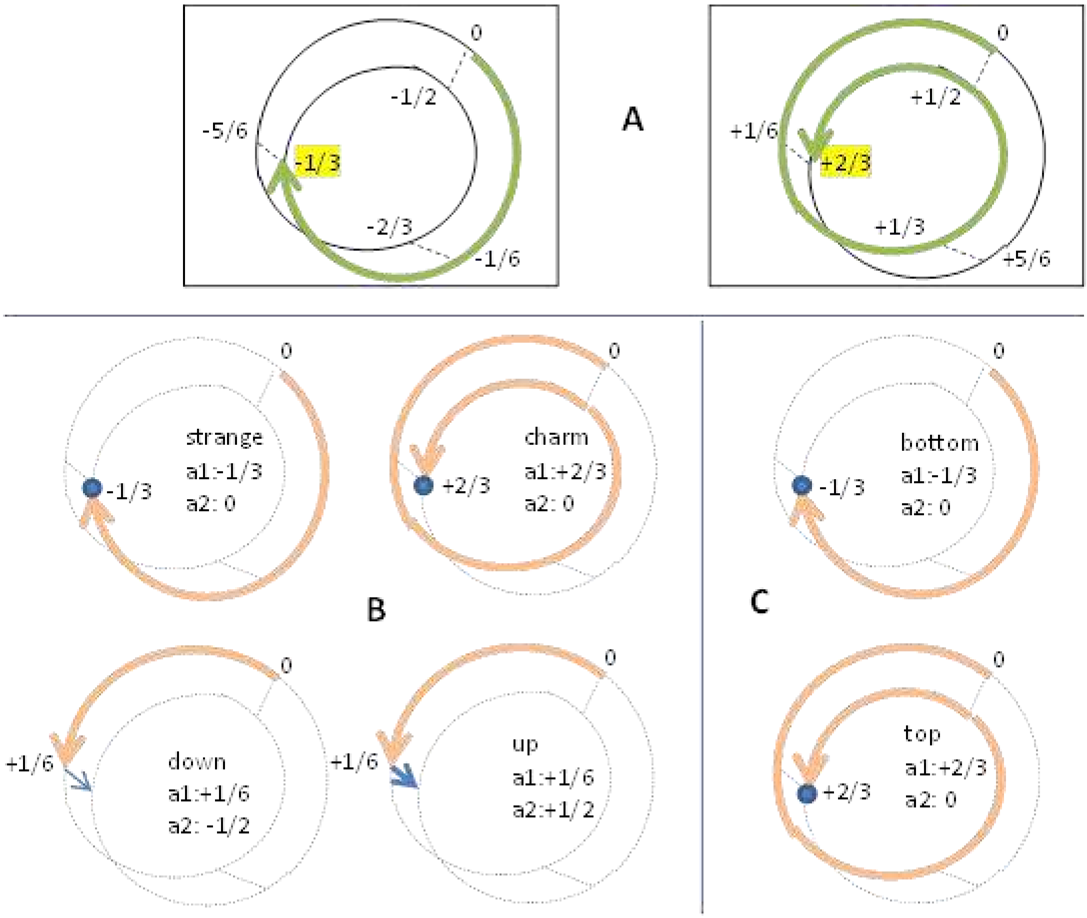


Figure 41. Depiction of the diametric replacements (quark duplets) of the a0 = -1/3 arc (prequark) and the a0 = +2/3 arc (alternative prequark) (green) (A) on the boundary of a Möbius strip (dashed double loop). The a1-bits (orange) and a2-bits (blue) of the a0 = -1/3 arc (B) and the a0 = +2/3 arc (C). The ratio of the a1 arcs to the boundary line is indicated by a fraction written at the endpoint of a1 with the sign corresponding to the direction of a1. An a2-bit with a value of 0 is depicted as a blue dot, while the a2-bit with values -1/2 and +1/2 is represented by a thin and thick blue arrow, respectively.

The arcs of the a0 = 0 diametrical replacements were depicted similarly to the quark duplets **(Figure 42)**.

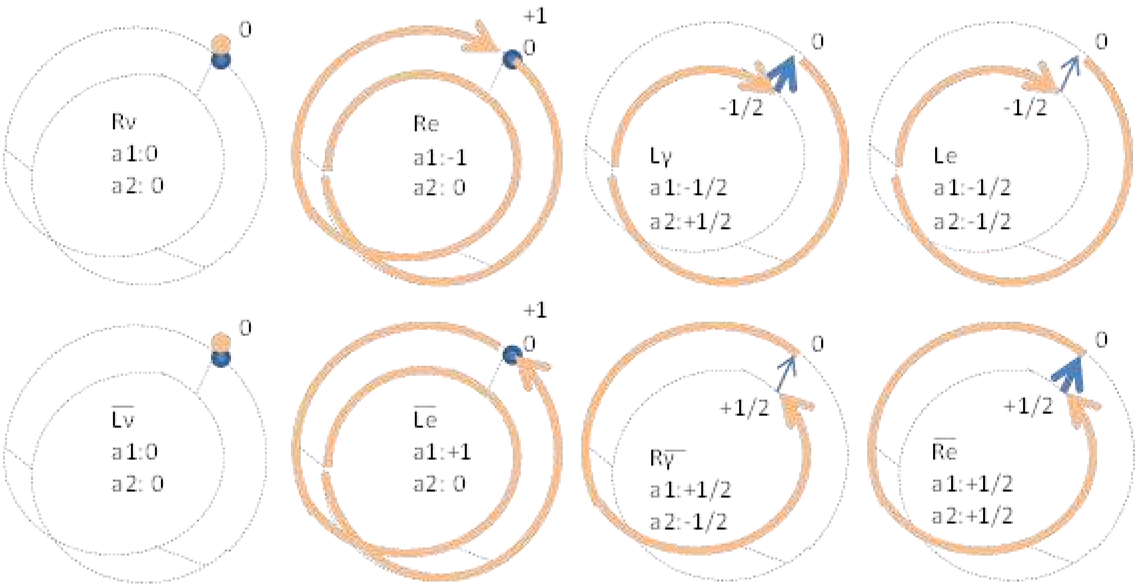


Figure 42. Representation of the diametric replacements of the a0 = 0 arc (lepton duplets and lepton duplets) on the edge of a Möbius strip (dashed double loop). The a1 (orange) and a2 (blue) bits of the a0 = 0 arc. An a-bit with a value of 0 is represented by a blue dot, while a2-bits with values of -1/2 and +1/2 are represented by thin and thick blue arrows, respectively.

The diametric replacement corresponds each fermion/fermion duplet in the fermion table **(Figure 30)** to arcs running to the three spokes of a Möbius strip.

For the lepton duplets and lepton duplets, there is only a single spoke, so the arcs corresponding to their quantum numbers end on the same spoke. In contrast, the quark duplets and quark duplets are replacements for two opposing a0 arcs, which run to two different spokes.

The curves running on the surface of the torus perform toroidal and poloidal rotations.

According to the rule of diametric replacement, the tandem pair of arcs a1 + a2 runs to the point [x, fc(x)], where [x, fc(x)] = [-1/3, +2/3] if x = -1/3, and [x, fc(x)] = [0, +1] if x = 0.

The endpoints of the two arcs running to the first endpoint of the spoke are indeed at the [x, fc(x)] point, so they reach their target without further addition, making the arc corresponding to them a2 = 0.

However, the two arcs running to the second endpoint of the spoke run to the [x, fc(x)] point only if we supplement them with an arc a2 = ±1/2, which represents a ±1/2 turn along the poloidal circle of the torus **(Figure 40)**.

The seemingly arbitrary rules of diametric replacement thus express the properties of the Möbius strip and torus space. The a1-bit and a2-bit of fermion/fermion duplets are the toroidal and poloidal turns of the arcs, and correspond to the Y'w and Iw weak charges of the fermions.

Four arcs running to the two end points of a spoke are the a1 arcs (a1-bits) of the diametric replacement.

The bidirectional coordinates (x and fc(x)) of the spoke's first endpoint are the $Y'w_R$ quantum numbers of the R-fermions.

The bidirectional coordinates of the spoke's second endpoint ((x + fc(x)+fd (x + fc(x))) and (x + fd(x))) are the $Y'w_L$ quantum numbers of the L-fermions **(Table 16)**, of which point 2 of the diametric replacement rule excludes the (x + fd(x)) value.

The 0-arcs running from the first endpoint to its first endpoint are the Iw_R quantum numbers of the R-fermions, while the arcs running from the second endpoint of the spoke to the first endpoint are the Iw_L quantum numbers of the L-fermions.

The electric charge of fermion/fermion duplets Q_E is the sum of the toroidal (a1) and poloidal half-turn (a2) rotations, Q_E = Y'w + Iw.

By taking two complete turns along the edge of a Möbius strip, one returns to the starting point. If one full rotation is chosen as the unit, the hyper charge (Y) values commonly cited in the literature are obtained. However, if the complete traversal of the Möbius strip edge is taken as the unit, then we work with Y' = Y/2, which correspond to the a1 arcs of the diametric replacement.

The a2 values (-1/2, 0, +1/2) are independent of the chosen unit of toroidal rotation and are equal to the isospin (I) values given in the literature.

Note 1: The length/width ratio of the Möbius strip cannot be smaller than √3 **(2)**, a visual demonstration of which can be seen in a video: Minimum Ratio for a Möbius Strip https://youtu.be/TesruiUiZuI?si=8xuALaiiTZB9kMM4.

The generative graph of the fermion/fermion duplets

The relationships determining the planar arrangement of the arrowheads of fermion/fermion crosses **(Figure 13 and Figure 14)** are vividly illustrated by the Möbius strip **(Figure 41 and Figure 42)**. On the edge line of the strip, the arc pairs a1, a2, leading from the 0 starting point of the prime arc and anti-prime arc to the ±1/3 endpoint, or returning to the starting point, correspond in the same way to the quantum numbers of fermions/fermions as the coordinates of the arrowheads of the crosses. Thus, they represent each other.

The starting point and endpoint of the primordial arc/anti-primordial arc and their diametric points (coordinates ±1/2 and ±1/6) are the connecting points of the a1, a2 arcs. In the case of fermion/fermion crosses, these ±1/2 and ±1/6 values are the shift terms of the primordial cross.

The fermion/fermion duplets depicted along the edge of a Möbius strip running on the torus surface **(Figure 41, Figure 42)** also correspond to all possible configurations of an arcs with ±1/3 toroidal and ±1/2 poloidal turns. This is a curve whose ±1/3 toroidal rotation is the rotation around the central axis of the Möbius strip. That is, the diametric replacements of the primordial arc and anti-primordial arc are the states of the primordial arc and anti-primordial arc on the Möbius strip.

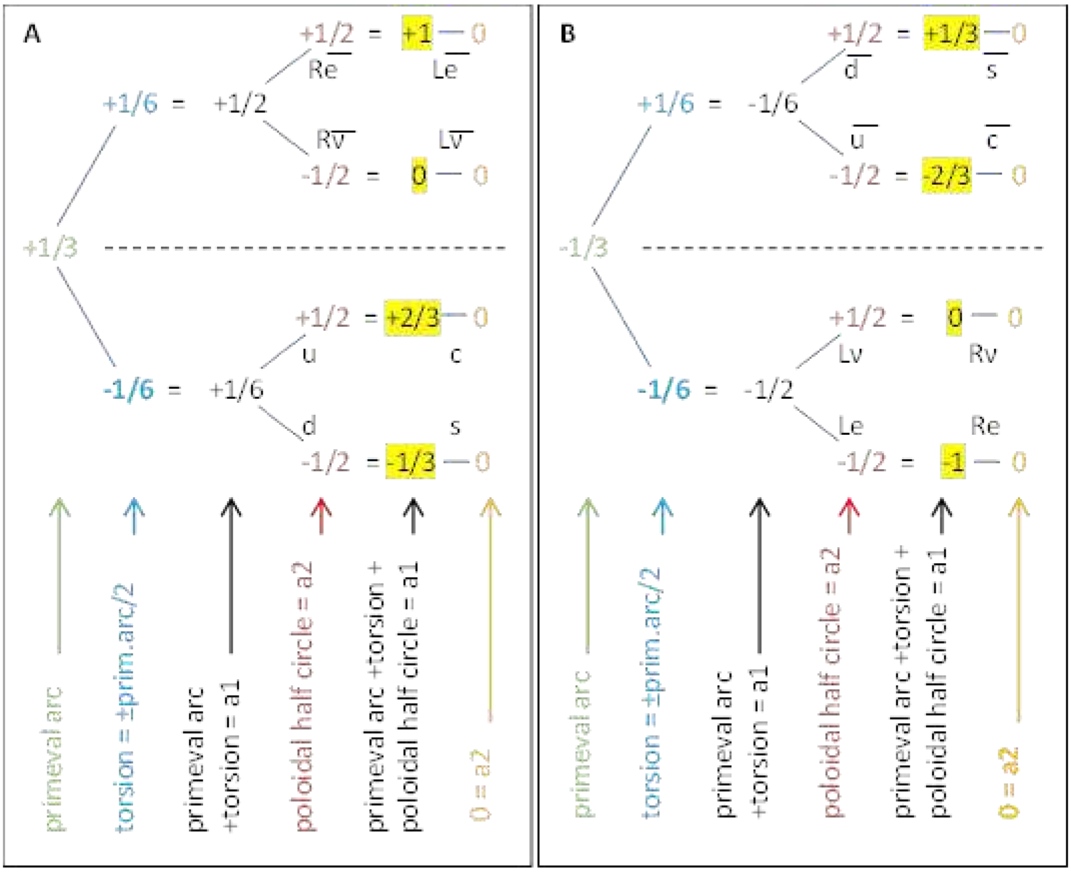
To see this, consider that an arc with a ±1/3 toroidal turn also has a torsional rotation of (±1/3)/2 = ±1/6 due to the torsion of the Möbius strip. In the topology of the Möbius strip, the ±1/3 and the diametric ±1/6 values are diametric alternatives **(Figure 39/C, D)**. The sign of the torsion is determined by the chirality of the Möbius strip.

For the two possible directions of toroidal rotation (±1/3), there can be two types of torsion settings (±1/6), which result in four sums of the toroidal + torsional rotations: (-1/3 -1/6) = -1/2, (-1/3 +1/6) = -1/6, (+1/3 -1/6) = +1/6, (+1/3 +1/6) = +1/2.

Each of these is associated with an additional ±1/2 poloidal turn, as well as a value 0 corresponding to the ±1/2 diametric value. This also forces us to give one more degree of freedom to the ±1/3 primordial arc, and later to provide the option to choose among three Möbius strips.

The ±1/3, ±1/6, ±1/2, 0 rotations are the dimensions of the configuration space, and the individual configurations correspond to fermion/fermion duplets. The configurations (fermion/fermion duplets) are characterized by the sums ±1/3 ±1/6 ±1/2 + 0, regarded as quantum numbers.

The partial sums of the tree-graph structured chain-additions are shown on the generative graph of the fermion/fermion duplets **(Figure 43)**. In the figure, we have combined the 0 root node of the graph and the first level containing the 0 ±1/3 addition, showing only the result of the addition and the subsequent levels of the graph.



C

Fermion/ $\overline{\text{fermion}}$	Generative code	Fermion/ $\overline{\text{fermion}}$	Generative code
$\overline{Re}$	+2 +1 +3 = +6	$\overline{Le}$	+2 +1 +3 +0 = +6′
$R\overline{v}$	+2 +1 -3 = -0	$L\overline{v}$	+2 +1 -3 +0 = -0′
u	+2 -1 +3 = +4	c	+2 -1 +3 +0 = +4′
d	+2 -1 -3 = -2	s	+2 -1 -3 +0 = -2′
$\overline{d}$	-2 +1 +3 = +2	$\overline{s}$	-2 +1 +3 +0 = +2′
$\overline{u}$	-2 +1 -3 = -4	$\overline{c}$	-2 +1 -3 +0 = -4′
Lv	-2 -1 +3 = +0	Rv	-2 -1 +3 +0 = +0′
Le	-2 -1 -3 = -6	Re	-2 -1 -3 +0 = -6′

Figure 43. The generative graph of fermion/fermion duplets. The addition operation is indicated by the lines connecting the members. The ±1/3 primordial arcs are green, the ±1/6 torsional members are blue, the ±1/2 poloidal semicircles are red, and ±0 are orange. The addition sign is replaced by the line connecting the members. On the branches of the generative graph, the terms of the last two addition indicate the quantum numbers of the L and R-handed variants of the fermion/ fermion duplets. The black numbers are the a1-bits (Y´w quantum numbers) of the duplets. The Y´w quantum numbers of the R variants highlighted in yellow. The associated (line-connected) numbers are the a2-bits (Iw quantum numbers) of the duplets. The figure shows only the result of the initial step (0 ±1/3 = ±1/3) of chain addition, at the root of the graph. The generative nomenclature of fermion/fermion duplets (C). The codes provide sixfold values of the members, thus indicating the number of U_{SD} units.

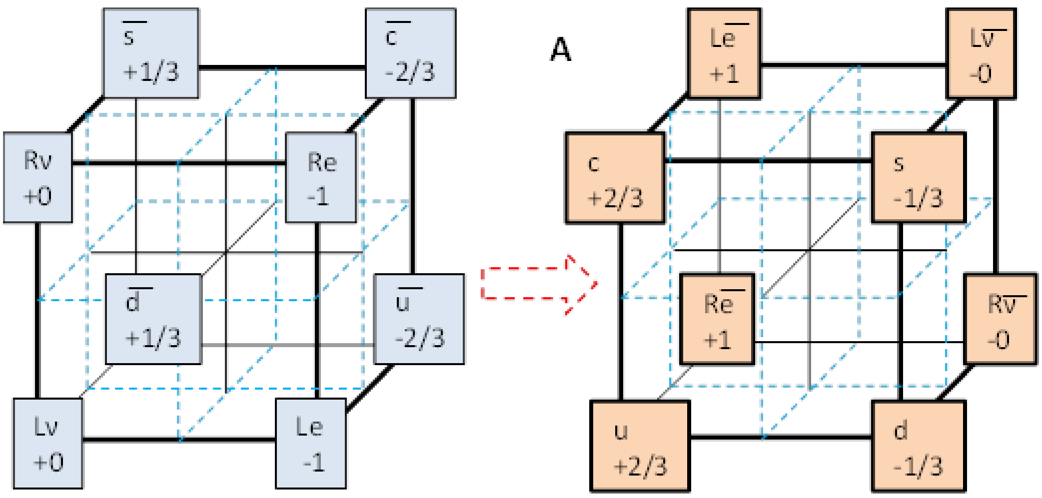
On the generative graph, the quantum numbers of the fermion table duplets are produced, and distinctions such as matter/antimatter, quark/lepton, down/up type, e/ν, generation 1/2, and good/bad chirality become meaningful.
On the branches of the generative graph, the last two summand terms represent the quantum numbers of the L and R variants of the fermion/fermion duplets.
The generative graph in this form is not complete, as indicated by the fact that it is not a complete binary tree graph. At its top level, it does not branch, meaning that only +0 appears instead of ±0. Expanding to the 6-quark model will eliminate this irregularity by incorporating the bottom quark duplet and the top quark duplet, which can also be interpreted as the possibility of choosing between three interconnected Möbius strips.

The nomenclature of the fermion/fermion duplets

To name the fermion/ fermion duplets and to encode the duplets, we introduce a generative nomenclature **(Figure 43/C)**.
The code of each fermion/ fermion duplet is one of the possible values of the sum ±1/3 ±1/6 ±1/2 ±0, formed from the coordinates of the starting point (0) and the endpoint (1/3) of the original arc, as well as their diametrically opposite points (1/2 and 1/6).
The signs of the four elements of the code can be read from the generative graph **(Figure 43/A, B)**, or they can be generated based on the properties of the duplet as follows.
In the code of fermion duplets, the second element (1/6) has a negative sign, while in the code of anti-fermion duplets it is positive. In the code of lepton duplets, the signs of the first (1/3) and second (1/6) elements are the same, whereas in quark duplets they are opposite.
The third component (1/2) has a negative sign in the code of up-type quarks and L, R electrons, and a positive sign in the code of down-type quarks and L, R neutrinos. In the codes of the anti-duplets, the signs are opposite.
The fourth component (0) contributes only to the code of second-generation quark/quark duplets and wrong-handed lepton/lepton duplets, which is why their code consists of four components. The codes of the other duplets consist of three components.
The fourth component of the code (0) does not change the total sum, therefore two codes with equal sums are distinguished by an apostrophe (') placed after the value of the code extended with 0.
The duplet code includes the duplet's Y'w and Iw quantum numbers as well as its electric charge. The Iw quantum number of the duplets is the last component of the code, while the Y'w quantum number is the sum of the components preceding the last one. The code components have values of 0 U_{SD}, ±1 U_{SD}, ±2 U_{SD}, ±3 U_{SD}, therefore in the generative code, by using values six times larger, we only need to apply addition and subtraction of the integers 0, 1, 2, 3 **(Figure 43/C)**.
In this case, the values of the code are even numbers, so we can assign the next odd number to the code value marked with an apostrophe.

The generative 4D cuboid of the fermion/fermion duplets

Based on the generative graph, we can construct the generative cube of the fermion/ fermion duplets, whose extent is ±1/3, ±1/6, ±1/2, ±0 (that is, the signed numbers 0, 1, 2, 3 multiplied by 1/6) in four dimensions **(Figure 44)**. The 4D generative cube is similar to the 4D q-cube of fermion/ fermion duplets **(Figure 24 and Figure 44)**. They differ in that in the generative cube, the origin has been moved to the center of the cube.
For comparison, we placed the 4D q-cube alongside the generative cube. For clarity, we represented the two 3D subspaces of the 4D cubes separately.
In the fourth dimension, the two 3D generative cubes correspond to the values -1/3 and +1/3. We can imagine this value written at the origin of the cubes. To this base value is added the extensions corresponding to the other dimensions.
The coordinate system of the 4D generative cube is identical to the α, β coordinate system of the quark chart extended to four dimensions, where one axis shows the values Γ1*α and the other axis shows Γ2*β. The chart's coordinate system is located in the horizontal middle plane of the two 3D subspaces **(Figure 44/A, B)**.
Thus, the framework of the 4D cube simultaneously illustrates the relationships between fermion/fermion a-bits, duplets, and the charts.



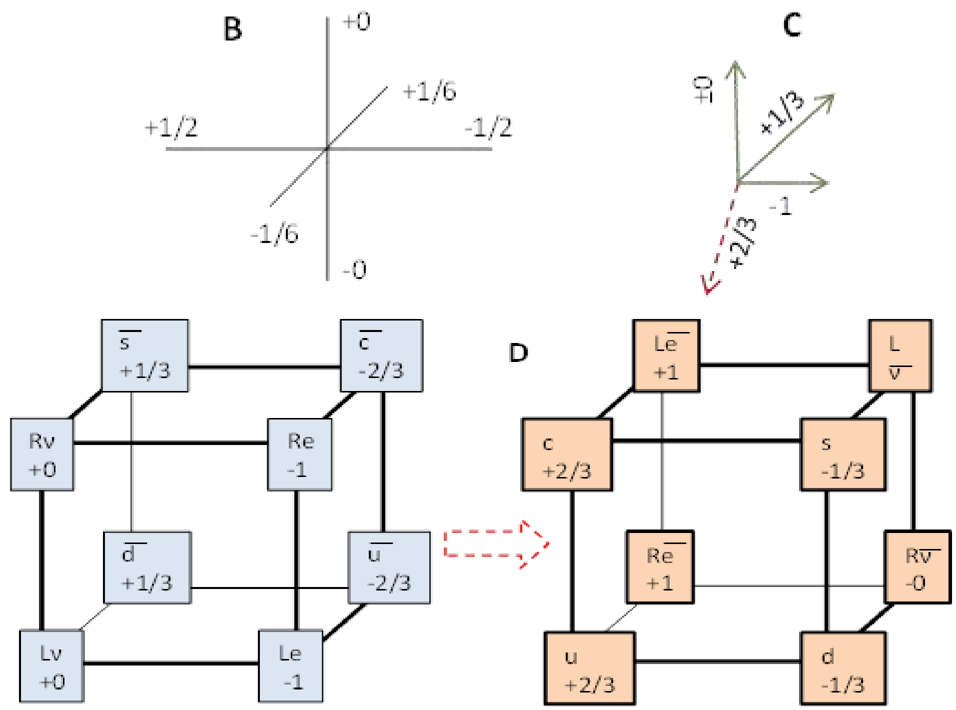


Figure 44. The 4D generative cube (A) with its coordinate system (B) and the 4D q-cube (D) with its coordinate system (C) of the fermion duplets. The red block arrow indicates the connection of the two 3D subspaces in the fourth dimension.

$2Q_L$ and $2Q_R$ quantum numbers of the fermion/fermion duplets

The Möbius strip is also a good model for the weak $2Qw_R$ and $2Qw_L$ quantum numbers of fermions.

The Yw', Iw quantum number pairs are represented by the a1, a2 arc pair, which is generated by the diametric replacement of a0, that is, it leads to the first endpoint of the quark spoke or the lepton spoke.

In contrast, the $2Qw_R$ and $2Qw_L$ quantum numbers are represented by an a1 arc, which also leads to one endpoint of the quark spoke or the lepton spoke, but this point can be either endpoint of the spoke. Therefore, the spokes do not have a distinguished endpoint, so there is no need for a second arc leading from the subordinate endpoint to the dominant endpoint, thus corresponding to an a2 quantum number.

At one end of the spoke, the arcs x and (x + fc(x)) lead, and at the other end of the spoke, the arcs (x + fd(x)) and (x + fc(x)) + fd(x + fc(x)) lead. The four coordinate values at x = -1/3 correspond to the $2Qw_R$ and $2Qw_L$ quantum numbers of quarks, and at x = 0 they correspond to the $2Qw_R$ and $2Qw_L$ quantum numbers of leptons **(Figure 45** and **Table 17/d, e column)**.

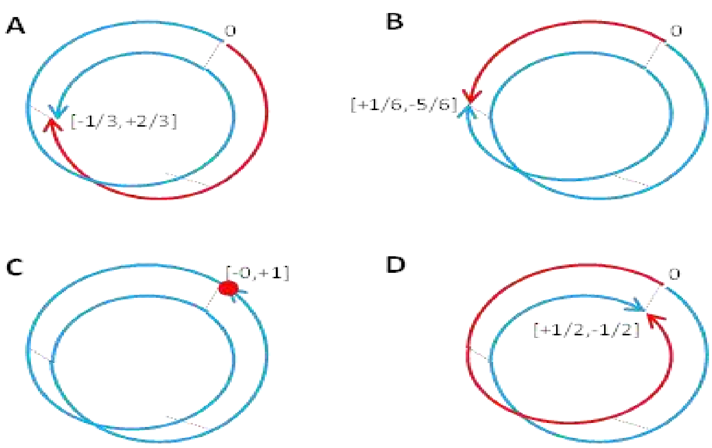


Figure 45. Representation of the quantum numbers 2QL and 2QR of fermion duplets on the edge of the Möbius strip.
Arcs representing the charges of up-type quark duplets 2QL = +2/3 (blue) and 2QR = -1/3 (red) (A), as well as the charges of down-type quark duplets 2QL = -5/6 (blue) and 2QR = +1/6 (red) (B). The arcs start from point 0 and meet at the endpoint of the quark spoke.
By adding +1/3 to the coordinates of the arcs representing the quark duplets, we obtain the 2QL and 2QR values of the lepton duplets. The arcs start from point 0 and meet at the end of the lepton spoke. Electron: 2QL = +1 (blue) and 2QR = -0 (red) (C); neutrino: 2QL = -1/2 (blue) and 2QR = +1/2 (red) (D).
The + and – coordinates of the meeting point are given in square brackets. The coordinate of the arc endpoint is determined by the ratio of the arc and the edge line, with a sign depending on the direction. The spokes of the Möbius strip (double loop) are dashed lines.

The bidirectional coordinates of one end of the spoke are x and (x + fc(x)) for the up-type quarks and the e, μ, τ quantum numbers, while (x + fd(x)) and (x + fc(x)) + fd(x + fc(x)) correspond to the down-type quarks and the neutrino quantum numbers.

Taking fermion duplets into account, at both ends of each spoke, two alternative arcs meet, if we consider the ±0_±1 spoke as two coinciding spokes, namely the +0_-1 and -0_+1 spokes. The $2Qw_L$ and $2Qw_R$ quantum numbers are in a one-to-one correspondence with the arcs leading to the spokes. Both the magnitude and the sign of the arcs match the quantum numbers.

Fer-rmion	Strong a		b	Weak c	d	e	f	g	h	i
	Y'	I	Q_E	Q_W	R $2Q_R$	L $2Q_L$	R $Y_W{}'$	L $Y_W{}'$	R I_W	L I_W
d s (b)	+1/6 -1/3 (-1/3	-1/2 0 0)	-1/3	-2/3	+1/6	-5/6	**-1/3**	+1/6	0	-1/2
u c (t)	+1/6 +2/3 (+2/3	+1/2 0 0)	+2/3	+1/3	-1/3	+2/3	**+2/3**	+1/6	0	+1/2
e μ τ	0		-1	-0	+1/2	-1/2	**-1**	-1/2	0	-1/2
νe νμ ντ	0		+0	+1	-0	+1	**+0**	-1/2	0	+1/2

Table 17. Quantum numbers characterizing the strong and weak interactions of fermions.

The quantum numbers $2Qw_R$ and $2Qw_L$ correspond to a single arc; diametric replacement does not play a role in their formation, and therefore the rules of diametric replacement do not apply, but their values also follow from the topology of the Möbius strip.

The arcs leading to the endpoints of the spokes of the Möbius strip are multiples of ±1/6 units. This unit, appearing previously as the translation term of fermion crosses, Γ2 coefficient, and pin, is called the **small diametric unit** (**U_{SD}**).

Every arc leading to a spoke can be decomposed into a sum of members containing copies 1, 2, and 3 of U_{SD}, that is, members of size ±1/6, ±1/3, ±1/2, where each member appears at most once, and each member must be followed by a member whose absolute value is 1 greater than its own.

Every integer $-6 \leq n \leq +6$ has exactly one such partition that satisfies these requirements **(Figure 46/B)**, provided that +0 and -0 are distinguished.

Thus, the coordinates of the arcs in the + and – directions, that is, the quantum numbers $2Qw_L$ and $2Qw_R$ of the fermion/fermion duplets, can be generated unambiguously even without the aid of the functions fc(x) and fd(x) **(Figure 46/A)**.

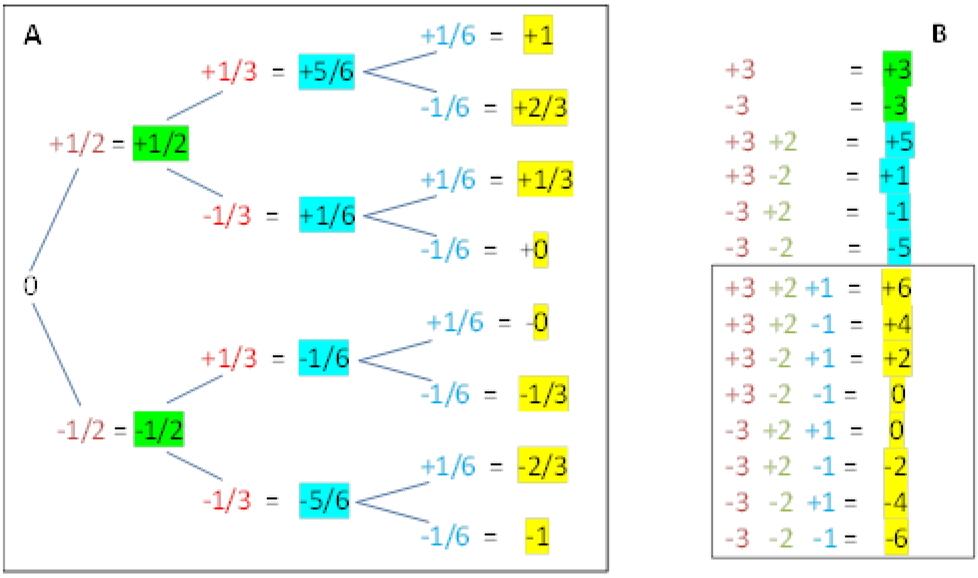


Figure 46. The generative graph producing the coordinates of the arcs leading to the spokes of the Möbius strip from the terms $\pm 3{*}U_{SD} = \pm 1/2$, $\pm 2{*}U_{SD} = \pm 1/3$, $\pm 1{*}U_{SD} = \pm 1/6$ (A), as well as the one-to-one correspondence of the arcs with the partitions of integers $-6 \leq$ integer $\leq +6$ (B).
The colored numbers of the generative graph are the terms ±1/2, ±1/3, ±1/6 added to the initial value 0. The addition operation is represented by a blue line connecting two values. The results of the additions are highlighted in green, blue, and yellow. The values highlighted in yellow are also the electric charges q of the fermion/fermion doublets.
The grouping of the partitions of integers $-6 \leq$ integer $\leq +6$ according to the number of terms ±1/2, ±1/3, ±1/6 (B) demonstrates that for every integer $-6 \leq$ integer $\leq +6$ (distinguishing between +0 and -0), there exists exactly one partition that satisfies the conditions described in the text. The partitions indicate six times the value of the terms, that is, the number of units in U_{SD} of the terms.

The arcs leading to the spokes are composed of members containing 1, 2, 3 U_{SD} units (with values of 1/6, 1/3, 1/2). The sign of the members determines the ide, opp, alt, com relationships between the arcs. In ide related arcs, the signs of the corresponding members are the same. In opp related arcs, the signs of the corresponding members are opposite.
In alt related arcs, only the member with the value 1/2 has a different sign. In com related arcs, only the member with the value 1/2 has the same sign.
We divided both the fermion/fermion duplets and the arcs leading to the spokes of the Möbius strip into members containing 1, 2, 3 U_{SD} units **(Figure 43, Figure 46)**. The divisions of the duplets into members form a special subset of the partitions of the arcs, which include the partitions containing all three members (±1, ±2, ±3) **(Figure 46/B framed partitions)**. At the same time, this special subset is made up of the partitions of the q electric charge of the fermion/fermion duplets (highlighted in yellow). The q charges are the final results of the chain additions.

The quantum numbers $2Qw_R$ and $2Qw_L$ indicate the strength of coupling to the bosons mediating the weak interaction, and their sum gives the weak charge of the fermions/fermions $Qw = 2Qw_L + 2Qw_R$ **(Table 17, column c)**.

The electric charge $Q_E = Y'w + Iw$ and the weak charge $Qw = 2Qw_L + 2Qw_R$ are complementary values (the sum of their absolute values is $|Q_E| + |Qw| = 1$).
Both the L and R fermions individually possess the same electric charge $Q_E = Y'w + Iw$, while the complementary Qw value is divided between the L variant and the R variant.
The Q_E and Qw quantum numbers of up-type quark duplets and down-type quark duplets are (-1/3)ide, (-1/3)com, (-1/3)alt, (-1/3)opp **(Figure 47/A)**, whereas the Q_E and Qw quantum numbers of electron-type lepton duplets and neutrino-type lepton duplets are (+0)ide, (+0)com, (+0)alt, (+0)opp **(Figure 47/B)**.

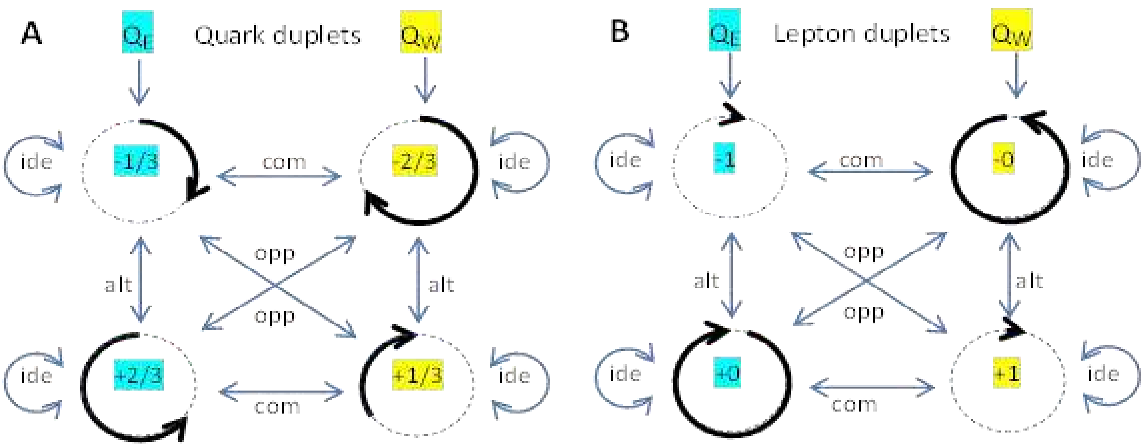


Figure 47. The ide, com, alt, opp relations between quantum numbers Q_E (turquoise) and Q_W (yellow) of fermion duplets.

Dream World factory

The physical names of the characteristics of duplets were adopted without considering other physical characteristics associated with them, even though quantum numbers are associated with characteristic values of mass, lifetime, and frequency, and they participate unevenly in the structure of the world.

In the large-scale distribution of matter, normal matter is conspicuously dominant compared to antimatter, which we encounter only in cosmic radiation and at high energy ranges in accelerators. For some quantum numbers, it is not even known whether they can be linked to independent entities.

In the pseudo-universe of the torus, this type of well-defined disproportion can occur by influencing the generative dimensions of fermion/fermion duplets. If we want to model our real world on the torus, we have to give torsion a distinguished direction, because a -1/6 torsion leads to quark and lepton duplets on the generative graph, and does not create anti-matter duplets.

The relatively stable atomic and molecular matter surrounding us consists predominantly of first-generation quarks and leptons of good handedness, or breaks down into these, and R leptons and L leptons do not participate in the reactions of the particles. The duplets corresponding to the commonly referred to particles are clearly distinguished on the generative graph from their special counterparts that appear at the end of the branches of the tree-graph.

To imitate our world, therefore, the fourth dimension of the duplets must be restricted in order to reduce or eliminate the duplets corresponding to extremely rarely occurring particles.

This means that on the generative cube of fermion duplets **(Figure 44)**, we do not distinguish between +0 and -0, thus eliminating the cube's extension in the -0 _ +0 direction, i.e., merging their top face with their bottom face. This transformation removes the difference between the two generations of quark duplets and unites the L lepton duplets with the corresponding R lepton duplets.

In this way, our model will tentatively be realistic, but this is an unjustified, arbitrary intervention in the autonomous development of our model so far.

However, it can be recorded as a result that anyone who does not find this world appealing can create one to their liking by adjusting the torsion and the other generative dimensions of fermions, at least on a torus.

The part of the physical universe with which we have paralleled our model is only a tiny fraction of the great unknown. Considering the mass of the total amount of matter, we are aware of more than 90% of it only through its gravitational effects and indirectly. We have no closer knowledge of the nature or carrier of this dark matter and energy, but the basic features of our model show certain similarities to the largest-scale matter distribution **(Figure 48)**.

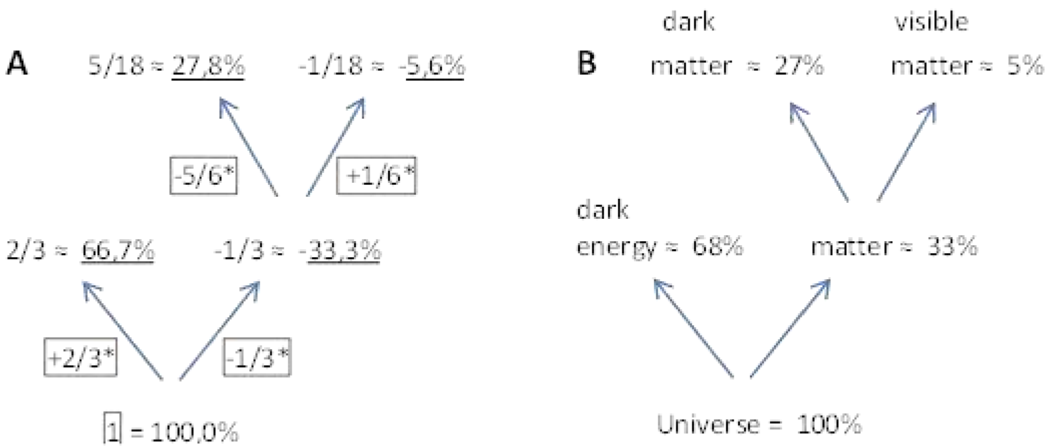


Figure 48. The application of the arcs running to the endpoint [x = -1/3, $f_c(x)$ = +2/3] and the diametrically opposite point [x + $f_d(x)$ = -5/6, x + $f_c(x)$ + $f_d(x + f_c(x))$ = +1/6] of the -1/3 arc, that is, the quantum numbers $2Qw_R$ and $2Qw_L$ of the quarks (framed), as a multiplier (A). The arrows point into the products. The distribution of the material of the universe (B).

The distribution represented by the -1/3 arc of the prequark that forms the basis of our model can be found in the primary distribution of the matter in the universe, separating into 33% matter and 68% dark energy. Comparing the total amount of matter to a circle, the -1/3 matter and +2/3 dark energy correspond to the prequark and the alternative prequark. The -1/3 and +2/3 multiplier ensures this **(Figure 48)**. Taking into account the accuracy of the matter quantity estimate, -33.3% and +68.7% is a good match.

The separation of the 33% matter into 5% visible matter and 27% dark matter corresponds on the circle to the two alternative arcs (+1/6 and -5/6) leading to the -1/3 diametrical point.

This suggests a structure similar to a cloverleaf, which is formed by the original circle creating 3 loops, and each loop is further divided into 3 parts, which are 1/9 portions of the original circle. Furthermore, all three loops form a double loop, that is, the boundary of a Möbius strip.

In this structure, the units of the original circle (1/3)*(1/6) = 1/18 and (1/3)*(5/6) = 5/18 can be interpreted, which correspond to the 1/6 and 5/6 arcs of the cloverleaf's leaves. The 1/18 and 5/18 correspond to 5.6% visible matter and 27.8% dark matter of the universe, which is in reasonable agreement with the realistically estimated 5% and 27% values.

So far in the paper, we have been moving along the perimeter of a leaflet of a clover leaf, applying the diametric replacement method to the -1/3 arc of it. The +1/6 arc of this circle appeared as the a1 arc of the replacements, while the -5/6 arc was excluded by the rules of replacements. Here, the +1/6 represents the ratio of visible matter, the -5/6 represents the ratio of invisible, dark matter in the total amount of matter.

The common origin of quantum numbers – Bruder Klaus reloaded

The fermion/fermion duplets and their quantum numbers were represented on the surface of a torus with arcs running from point 0 of the edge of a Möbius strip to the spokes. The two ends of the spokes were placed next to each other by knotting the unit circle.

If the circular line forms three double loops, then the values of every quantum number of both the elementary particles appearing in the paper and those not appearing are mapped onto a single point, concentrating at the center of a cloverleaf-shaped figure **(Figure 49)**.

On this six-times-knotted line, both the starting point and the endpoint of the arc corresponding to every quantum number of every elementary particle are at the center of the cloverleaf-shaped figure, and each appears in three variants due to the triple symmetry of the cloverleaf.

Each leaflet of the shape (one-third of the total line length) is a double loop and corresponds to the edge of a Möbius strip. The three double loops form the connected edges of three Möbius strips, which run on the surface of a 3-torus.

A 3-torus is created when the opposite faces of a cube are matched with each other, in the same way that opposite edges of a square were matched to create a 1-torus. The interior of a 3-torus would be similar to our 3D space, but no matter which direction one travels, they would return to the starting point.

Quoting Bruder Klaus again: '...just as they go out in divine power, so they also go in; they are one and undivided in eternal rule. That is what this figure means.' **(1)**.

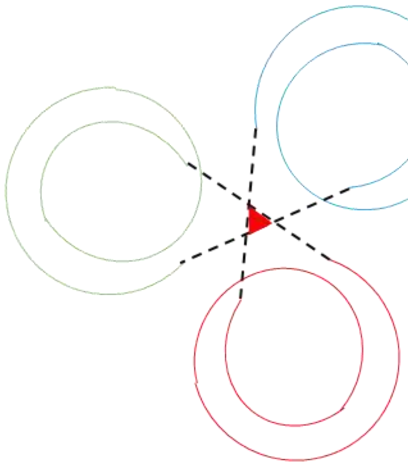


Figure 49. A sixfold twisted unit-length line, the connected boundary of three Möbius strips. The dashed lines indicate the connection of the three double loops. The center of the cloverleaf-shaped figure (red triangle) is the origin and the common endpoint of arcs of lengths 0, 1, -1, +1/2, -1/2, +1/3, -2/3, +2/3, -1/3, +1/6, -5/6, -1/6, +5/6.

Discussion

In the first part of the physical particle model of the Smart Dreamcatcher **(1)**, the properties of the basic elements of the SD-graph (mini SD-graph) were presented, and this time they have been displayed on a circle. We demonstrated that the topological properties of a circle imply the existence of entities (a-couples) that have no properties other than their characteristic numbers (a-bits); and these quantitative properties correspond to the properties of hyper charge, isospin, electric charge, weak hyper charge and weak isospin, weak charge, and chirality, which characterize the fermion and anti-fermion particles of the Standard Model. These entities are pairs of arcs (duplets) that are generated by the trivial rules of the diametric replacement procedure introduced here, by decomposing an arc of the circle (prequark).

Placing the quark/anti-quark duplets and lepton/anti-lepton duplets in the α, β coordinate system of the charts facilitates understanding the quantitative relationships among the quantum numbers (a-bits) of the duplets.

In the diametric replacement procedure, the point diametrically opposite on the circle plays a role similar to that of the points on the circle itself, from which we infer that the natural space of the quantum numbers is the boundary of a Möbius strip on the surface of a torus. In this space, the components generated by the diametric replacement procedure reflect the modeled physical reality in a necessarily true unity.

We have given names borrowed from particle physics, referring to subatomic analogies, to the structural elements and their quantitative characteristics that emerged during the autonomous development of our quantum number universe.

The quantum numbers were decomposed into simpler, illustrative constituents (Γ1, Γ2) which are necessary consequences of the diametric replacement procedure.

These components, which behave as elementary units and prove useful in the description of the model, as well as the family of reverse lepton duplets that appear as a logical part of the system, could not be identified with observed elementary particles.

The theoretical construction developed here has been scrutinized from multiple perspectives and integrated into a broader framework, some properties of which have also been discussed.

A necessary consequence of applying the diametric replacement procedure is the appearance of duplets corresponding to the bottom and top quarks, which we only touched upon here. Following the chronological development of physics, the illustration of the six-quark model will continue in the next part of the Smart Dreamcatcher physical particle model.

In the study, the structure created through diametric replacement was presented as a co-domain of a function, as a partitioning of integers, as structures in Euclidean space and as topological properties of surfaces, which only appear different due to the way they are described. The representations of the circle, chart, Möbius strip, torus, fermion cube, generative graph, fermion table, and SD-graph introduced here all serve as suitable tools for the didactic presentation of elementary particles.

We have pointed out that the concepts and thought patterns developed from our everyday experiences can also be useful in the study of elementary particles.

That is, the quantitative system describing the properties of elementary particles also applies to an independently generated pseudo universe.

As a result, this model can be used to visualize the relationships between the properties of elementary particles and the structure of the Standard Model. Furthermore, it also can help in string theory to find the appropriate topology.

Additionally, structural and logical analogies of the SD model can be found in seemingly distant areas, such as the organizational level of living matter. Examples of structures surprisingly similar to the helical form of the SD graph stretched on a cylinder are the 3_{10}-helix, α-helix and π-helix conformations of proteins, where nature forms SD graphs from the system of covalent and hydrogen bonds. (Adél N. Juhász personal communication).

The basic structure of amino acids, with three functional groups (forming the skeleton of proteins) arranged around the central carbon atom, corresponds to the hyper edges of the SD graph, while the simplest cyclic tripeptide serves as an analogue for a zone of the graph.

The coding of protein synthesis also allows recognition of the stable pathways of the SD graph. Any three of the four nucleotides in the code correspond to an amino acid, just as any three steps on the SD graph correspond to a baryon, provided that the hyperedges's traversal direction is fixed, meaning that anti-quark steps are not allowed on it.

Consequently, the SD graph also serves as a model for the structure of proteins and their synthesis, and for their shared mystery, thus the origin of chirality.

By further extending the model, we can form a visual concept of why we do not find more chemical elements on the Mendeleev table, and why similar ratios can be recognized at different levels of organization, in the spinning of galaxies and even in the distribution of matter on the largest scale in the universe.

At the same time, the diversity of these analogies raises the epistemological question of whether we are objectively modeling the world, or rather discovering from reality only what we can adapt to our own preformed mental templates, creating our own Universe "in our own image and likeness".

Afterword

The musical supplement (mp3 sound file at the end of the article), my dear friend Pauline Grundler's (Chicago, Illinois USA) AI-powered composition titled Mandala of Light, was inspired by reading the manuscript. Her valuable contribution reflects that there is a crossover between different areas of interest, and that the exploration of the micro-universe resonates regardless of one's original inclinations.

The Sanskrit-derived word mandala, chosen as the subtitle, means circle. It is semantically and perhaps even etymologically related to the word manas (mind). Mandalas are meditative, ritualistically created representations of aspects of the cosmos and states of consciousness. They hold great significance in Hindu and Buddhist cultures, but one can also encounter living traditions of mandala philosophy and its material manifestations in other cultural contexts.

Mandalas are often depictions based on symmetries and multiple reflections, and we do not know why their creation has such an intense effect on the spiritually engaged participant in the ritual. It is often reported as an inexplicable metamorphosis, a moment of recognition of the guiding motif hidden behind the multilayered relationships of the details, when we learn to see the same thing differently, to which the subconscious has not yet found an adequate expression in a world built from words.

The source of the fundamental but intangible revelation of those meditating on a transcendental circle, which goes beyond the limits of language, is unknown, and the Sanskrit language called it a mandala. When constructing our own mandala, we walk around a circle and return to where we started. The rest takes place in our minds, where cosmic connections take shape out of thin air, when reality shows its many faces at once.

We can dream of new paths and new dream worlds to which the paths lead. On the way, we can marvel at the tiny details or the simplicity of the greater whole.

In our mandala built from numbers **(Figure 50)** (which we have already encountered from a different perspective in **Figure 12**), we can see the complexity of a micro-world or the pattern of a curtain of monotonous number sequences in orange, blue, green image cutouts.

Sometimes the simplest things require the most complicated explanation, while other times the most complicated things have the simplest explanation, and often it's not clear which one is the case. The experience of the mandala is that there is no need to choose between them. Finally, the mandala can even be erased, because it is only about a simple circle, which can bring the experience to life again at any time.

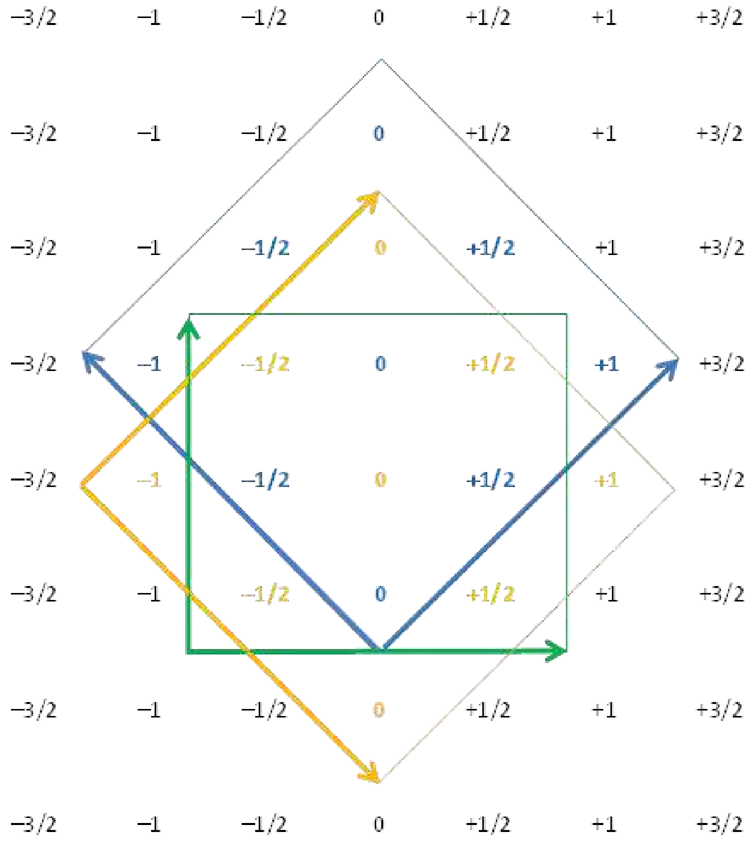


Figure 50. Our 'a-bit Mandala of The Circle', which is the function of **Figure 12** in a different interpretation, a cascade of numbers in a blue, orange, green cutout.

Acknowledgment

I am grateful to Mrs. Desiree Winkler (Zürich) for her attentive curiosity, wanting to know why I consider the world of elementary particles an incredibly exciting theme park, even though it is actually an alarmingly unknown and alien place. Without Desiree's encouraging curiosity, I would never have started to write down my little apologies, which at that time and place had no opportunity to be expressed: the magic is in the air, and all around us in the Universe, whose familiar and charming landscapes you usually call 'Mother Nature.'

To be honest, this is something different from the popular cuteness that the trend of Mother Earth or Mother Nature picks up (while maybe we're actually not all that curious about tiny things or about Mother Nature herself).

It shows the view that I see from my own window. The magic is there in both the living and non-living, in its beautiful landscapes and little details that we might come across while wandering through the fantastic lands of our own dream world.

Mandala Of Light .mp3

Reference

1. Juhász A, The „Smart Dreamcatcher" (SD) Physical Particle Model: I. Day-Dreams of the Hermit, Eur. J. Phys. Edu. (2022) Vol. 13:1, 28-43
2. Schwartz R. E, The optimal paper Moebius band, Ann. of Math. (2)201(1):291-305(January 2025). DOI: 10.4007/annals.2025.201.1.5